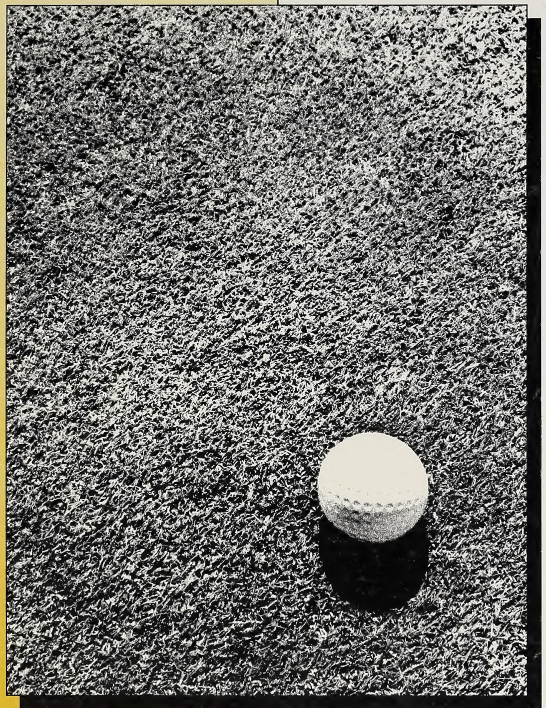
MATHEMATICS 8



MODULE 2

WHOLE NUMBERS AND INTERGERS

Distance Learning

Alberta EDUCATION

Mathematics 8

Module 2: Whole Numbers and Integers

MODULE BOOKLET

Mathematics 8
Student Module
Module 2
Whole Numbers and Integers

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Welcome to Module 2!

We hope you'll enjoy your study of ***Whole Numbers and Integers****.*

To make your learning a bit easier, a teacher will help guide you through the materials.

So whenever you see this icon,

turn on your audiocassette and listen.

CONTENTS AT A GLANCE

MODULE INTRODUCTION

What Lies Ahead

In the Module Introduction you will preview the components of this module and you will learn how this module is evaluated.

Working Together

In Modules 2 and 3 of Mathematics 8 you will be learning about number systems and operations.

You will be learning about different kinds of numbers.

- whole numbers
- integers
- decimal numbers
- fractions
- rational numbers

You will perform operations with different numbers, and you will learn some number theory.

You will find these modules helpful because in today's world you will encounter a wide variety of numbers. People, places, and things are numbered.

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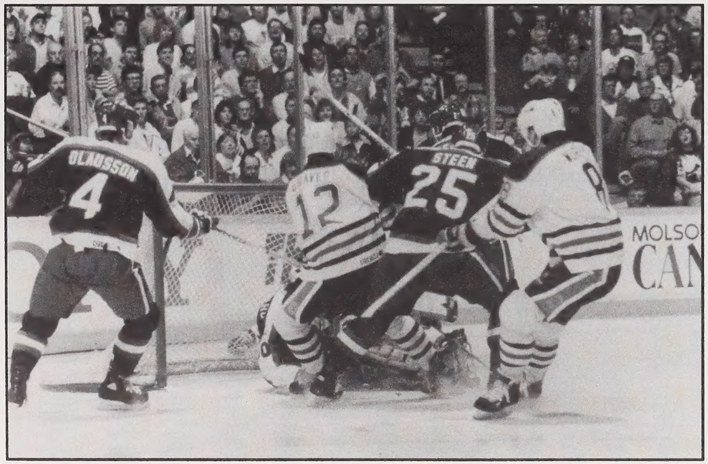

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Numbers represent places in a rank, scale, or ordered frame.

- The temperature is – 16°C.
- St. Valentine's Day is February 14.
- Her address is 5417 - 57 Avenue.
- Find the mathematics book under 512.3 in the library.

Numbers represent counts.

- There are three people in the Cox family.
- Jason has one sister.
- Anthony ate $5\frac{1}{2}$ chocolate bars.
- Loi has 585.26 dollars.

Numbers represent measures.

- Matthew is 14 years old.
- Jonathan is 166 cm tall.
- Ruth weighs 50.5 kg.

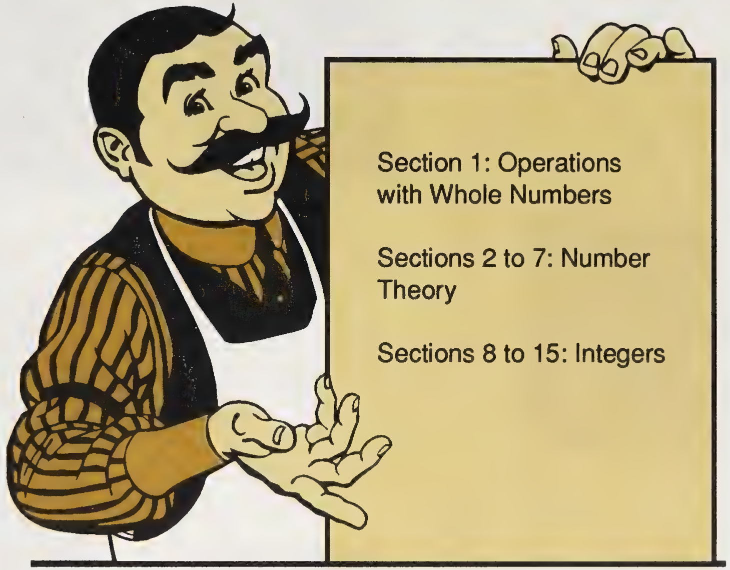
You will also learn that numbers have many different but equivalent forms. For example, the following are different forms of the same number.

- $3\frac{1}{2}, 3\frac{2}{4}, 3\frac{3}{6}$
- $\frac{7}{2}, \frac{14}{4}, \frac{21}{6}$
- 3.5, 3.50, 3.500

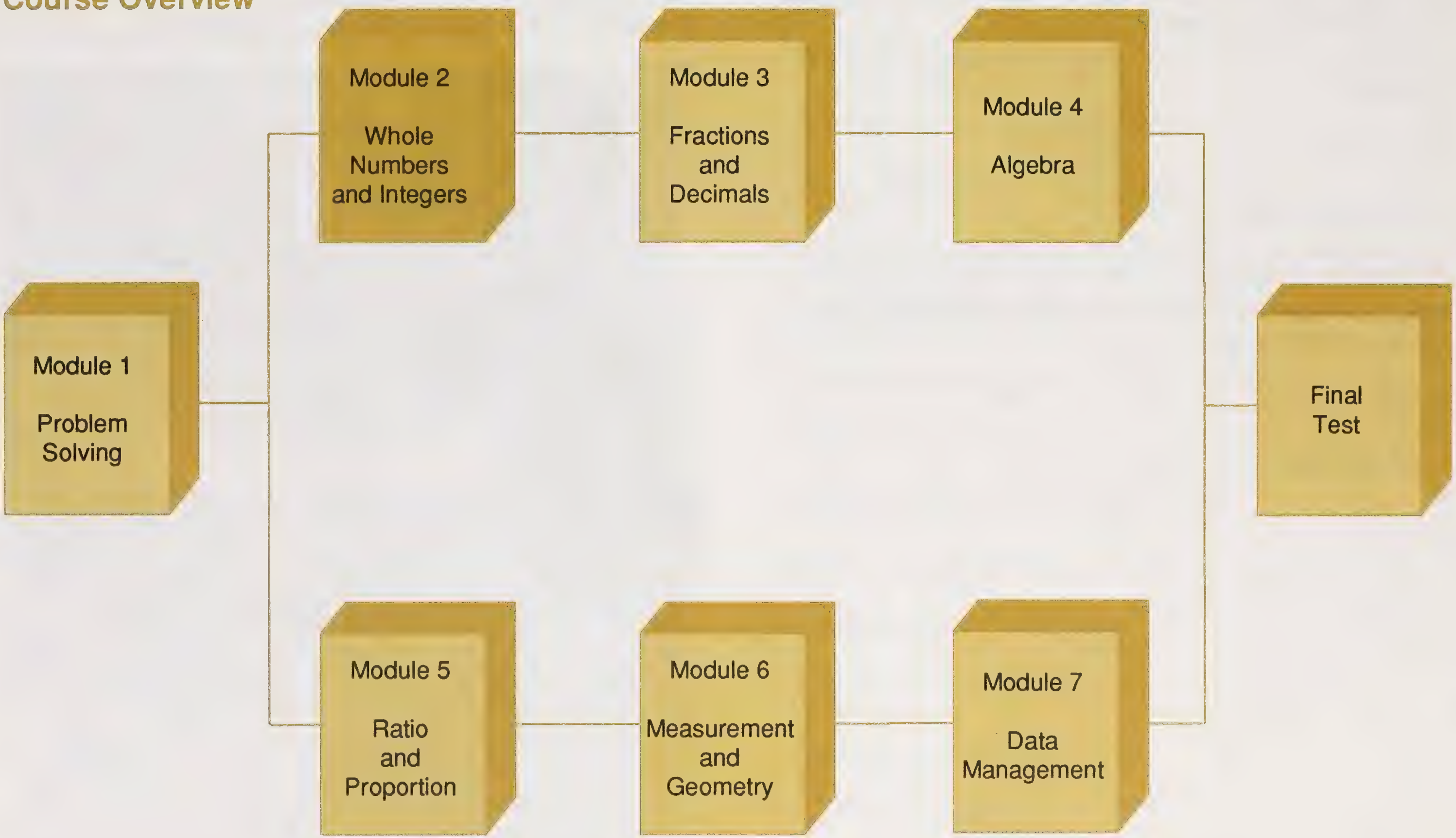
You will discover that some forms are easier to work with. For example, large numbers can be written in a more compact way.

- $125 = 5^3$
- $81\ 000\ 000\ 000 = 8.1 \times 10^{10}$

In Module 2 you will begin your study of number systems and operations. You will deal with whole numbers and integers. This is how the module is organized.

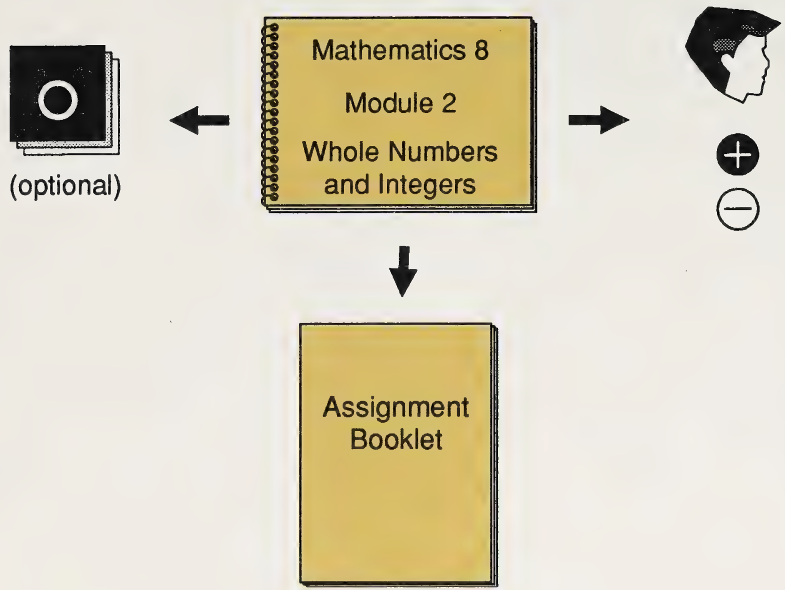


Course Overview



Mathematics 8 has seven modules and a final supervised test. This module booklet is part of Module 2.

Module 2 Components

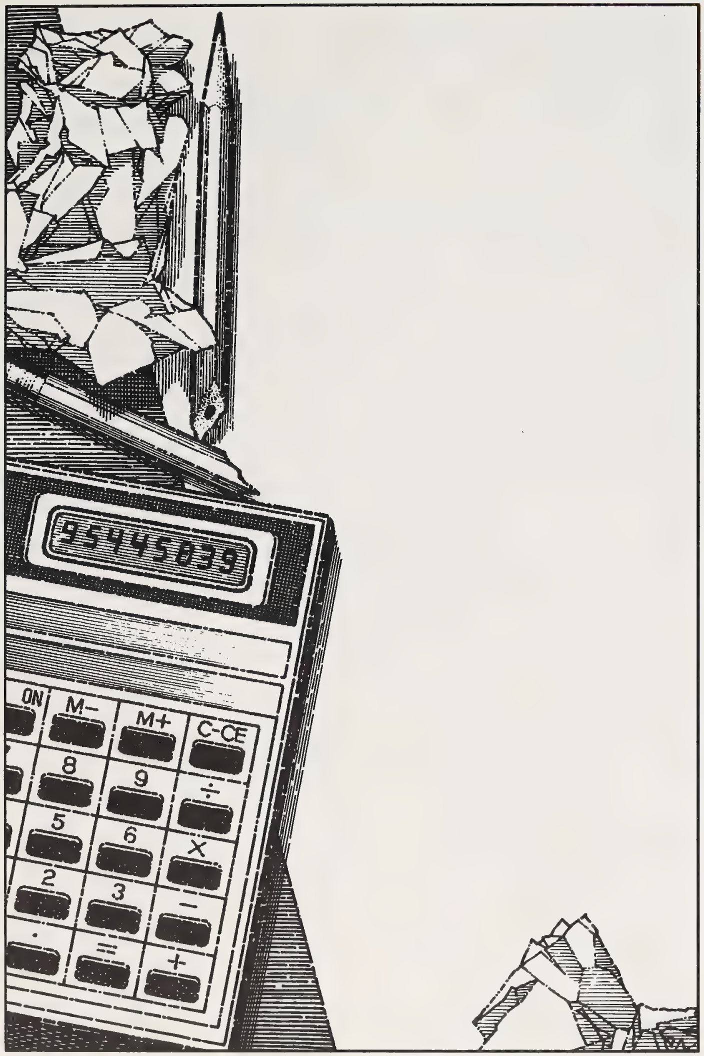


This module booklet will give you instruction and practice in the skills and mathematical words you are required to learn. It will also direct you to the other components of the module. The computer activities in this booklet are optional. There are print alternatives. You should see your learning facilitator to check your answers to the activities in this booklet. This booklet is not to be submitted for a grade.

Your mark on this module will be determined by your work in the Assignment Booklet.

Take time to preview this module booklet now.

PART ONE



Part One reviews operations with whole numbers. You will review how to estimate sums, differences, products, and quotients. You will also review how to find exact sums, differences, products, and quotients by using mental computation, paper and pencil, and a calculator. Finally, you will review the rules for order of operations.

KEEPING SKILLFUL

What Lies Ahead

In this section you will review these topics.

- operations
- order of operations

Working Together

To begin this module you will review your skills with whole numbers. This review will help you and your learning facilitator discover your strengths and weaknesses.

Review

Space for Your Work

1. Calculate the exact answers. Do not use a calculator.

a. $\begin{array}{r} 64 \\ +\,29 \\ \hline \end{array}$

b. $\begin{array}{r} 287 \\ +\,809 \\ \hline \end{array}$

c. $94 - 37$

d. $\begin{array}{r} 201 \\ -\,193 \\ \hline \end{array}$

e. $\begin{array}{r} 28 \\ \times\,19 \\ \hline \end{array}$

f. 37×68

g. $7\overline{)3310}$

h. $4395 \div 63$

2. Calculate each of the following mentally.

a. $529 + 356$

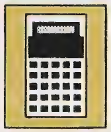
b. $872 - 356$

c. $8 + 6 + 2 + 5 + 4$

d. 32×40

e. $36\,000 \div 600$

Use a calculator to do Questions 3 to 5.



3. a. What is the total cost of the skateboard?

   b. How much change is left from $200?

Deck......$88 Wheels.........$40

Trucks...$32 Bearings.......$ 8

Space for Your Work

Space for Your Work

4. If Gloria's heart beats 69 times in one minute, how many times does it beat in one hour?

5. The Petersons travelled 570 km in six hours on the first day of their vacation. Find the distance they travelled in one hour.

6. Evaluate the following using paper and pencil.

   a. $29 - 5 \times 3$

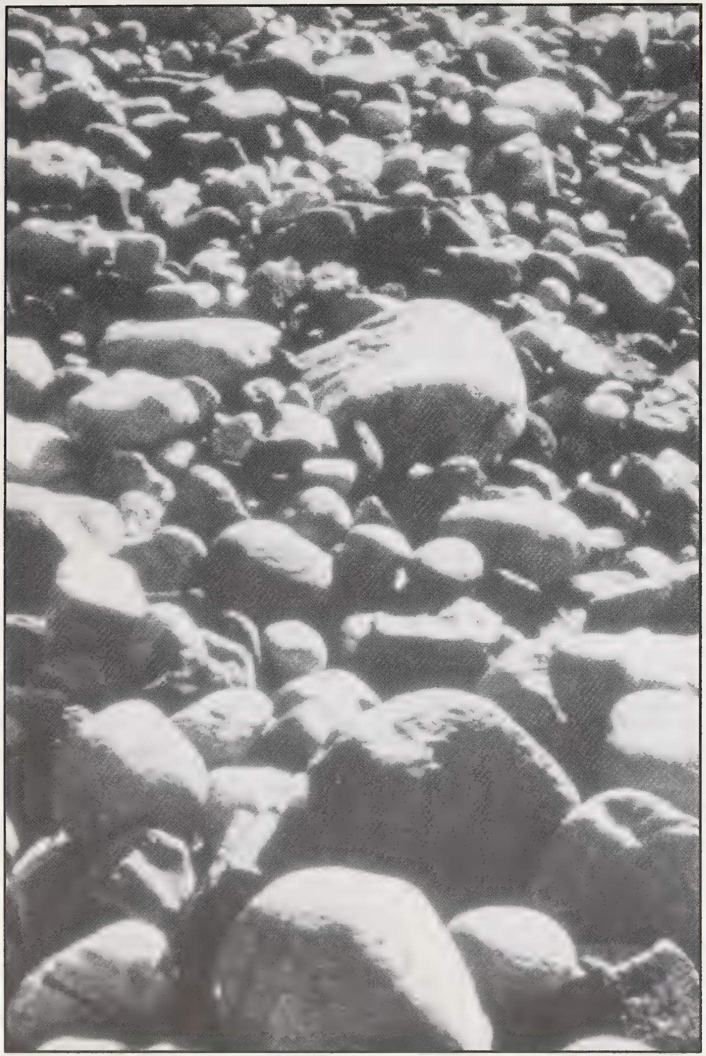
   b. $56 \div (10 + 6 - 8)$

   c. $7 + 7 \times 7 + 7$

   d. $\dfrac{3 \times 5 - 1}{16 \div 8}$

✓ See your learning facilitator to check your answers and to receive further instructions.

PART TWO



Part Two of this module deals with number theory. In Sections 2 to 7 you will study whole numbers and the relationships between numbers. You will learn about factors, multiples, powers, and scientific notation.

What Lies Ahead

This section will pretest the following skills.

- finding the factors of a number
- finding the common factors and the greatest common factor (GCF) of two or more numbers
- finding the multiples of a number
- finding the common multiples and the least common multiple (LCM) of two or more numbers
- expressing numbers in standard form as powers
- expressing powers as numbers in standard form
- expressing numbers in standard form as numbers in expanded form
- expressing numbers in expanded form as numbers in standard form
- expressing numbers in scientific notation

Working Together

The pretest in this section will help you and your learning facilitator to determine your strengths and weaknesses.

Pretest

Space for Your Work

1. a. List all the factors of 48, 56, and 80.

   b. Identify the common factors.

2. Find the greatest common factor (GCF) for each of the following pairs of numbers.

   a. 15 and 30

   b. 48 and 60

   c. 24 and 51

3. A bakery sells oatmeal cookies in two different package sizes. The price per cookie is a whole number and remains the same. One package sells for 50¢, the other for 80¢. What is the most that each cookie could cost?

4. List the first five multiples of 13.

5. Find three common multiples for each of the following pairs of numbers.

    a. 4 and 5

    b. 9 and 6

    c. 3 and 9

6. Find the least common multiple (LCM) for each of the following groups of numbers.

    a. 4 and 9

    b. 6 and 8

    c. 6, 9, and 10

Space for Your Work

Space for Your Work

7. Mark is practising for a marathon. He runs one lap of the practice course in four minutes. His younger brother, Monty, can run the same distance in seven minutes.

a. If they both start at the same time and keep running until they both cross the finish line together, how long will it take?

b. How many laps will each have run?

8. Express each of the following as a power.

a. $156 \times 156 \times 156 \times 156 \times 156$

b. $19 \times 19 \times 19 \times 19 \times 19 \times 19 \times 19 \times 19 \times 19$

9. Write each of these numbers in standard form.

   a. 15^2

   b. 9^4

   c. 10^5

10. Express each number as a power of 3.

   a. 27

   b. 243

11. Express each of the following standard numbers as a number in expanded form.

   a. 2 641 395

   b. 13 051 047

Space for Your Work

12. Express each of the following as a number in standard form.

a. $(3 \times 10^4) + (5 \times 10^3) + (7 \times 10^2) + (3 \times 1)$

b. $(5 \times 10^4) + (9 \times 10^2) + (8 \times 10^1)$

13. Express each of the following in scientific notation.

a. 12 000

b. 6 165 000

14. Express each of the following in standard form.

a. 2.13×10^3

b. 9.003×10^8

Space for Your Work

See your learning facilitator to check your answers and to receive further instructions.

GREATEST COMMON FACTORS

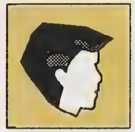
What Lies Ahead

In this section you will learn these skills.

- finding the factors of a number
- finding the greatest common factor (GCF) of two or more numbers
- using the GCF in problem solving

In this section you will use these words.

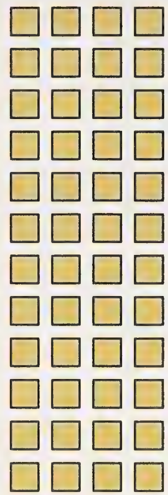
- factor
- prime factor
- common factor
- greatest common factor
- composite numbers
- prime numbers
- relatively prime numbers
- proper factor



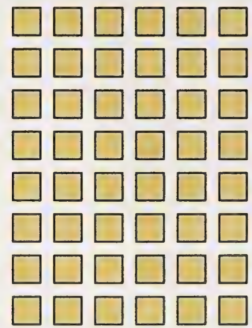
Working Together

Do you remember what a factor is?

A **factor** is a number which can be used to make a product. For example, 12 and 4 are factors of 48 because $12 \times 4 = 48$. Two other factors of 48 are 8 and 6 because $8 \times 6 = 48$.



12 groups of 4



8 groups of 6

Can you list all the factors of 48?

One way to list all the factors of 48 is to begin with 1 and test each consecutive number to see if it is a factor of 48.

Begin with the left column and work down. Then move to the right column and work up.

48		
1	48	
2	24	
3	16	
4	12	
~~5~~		5 is not a factor.
6	8	
~~7~~		7 is not a factor.
~~8~~	~~8~~	8 has already been mentioned.

The factors of 48 are

1, 2, 3, 4, 6, 8, 12, 16, 24, and 48.

Do you remember what a composite number and a prime number are?

A **composite number** has more than two factors. For example, 48 is a composite number because it has more than two factors.

A **prime number** has only two factors (1 and itself). For example, 2 and 3 are prime numbers.

Do you remember what a prime factor is?

Prime factors are simply factors which are also prime numbers. For example, 2 and 3 are prime factors of 48. The other factors of 48 (1, 4, 6, 8, 12, 16, and 24) are not prime factors.

Note

Remember that 1 is not a prime factor. To be a prime factor a number must have two factors, 1 and itself. 1 has only one factor, 1.

Do you remember what prime factorization is?

Prime factorization is the expression of a number as the product of its prime numbers.

Example: Give the prime factorization of 48.

Solution

There are two ways to find the prime factorization of a number.

Method 1: Dividing by Prime Factors

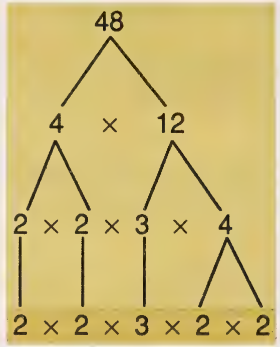
$$\begin{array}{r|l} 2 & 48 \\ 2 & 24 \\ 2 & 12 \\ 2 & 6 \\ 3 & 3 \end{array}$$

Divide the number and each succeeding quotient by a prime number.

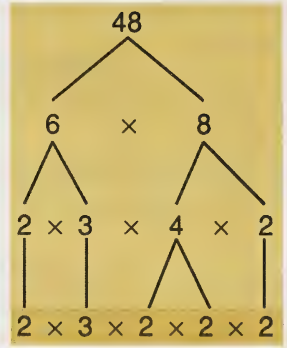
This is the prime factorization of 48.

$48 = 2 \times 2 \times 2 \times 2 \times 3$

Method 2: Using a Factor Tree



or



$$48 = 2 \times 2 \times 3 \times 2 \times 2$$

or

$$48 = 2 \times 3 \times 2 \times 2 \times 2$$

It does not matter which method you use; the prime factorization will always be the same for a number.

Note

It is customary to write prime factors in order from smallest to largest.

$$48 = 2 \times 2 \times 2 \times 2 \times 3$$

Games

Here are two games that involve prime numbers and prime factors.

How to Play *Place Roll*

Step 1: Choose the starting player by rolling the dice. The player who rolls the highest number goes first.

Step 2: The starting player rolls the dice and forms a number with the top two faces. The larger number represents the tens position and the smaller number represents the ones position.

Example:  = 43

Step 3: The player then puts a disc in one box on the playing board that contains a factor of that number.

Example:  = 32
The player can cover 8, 4, 2, or 16.

Step 4: If the player rolls a double, such as 33, 22, or 11, the turn is lost.

Step 5: If the player rolls a prime number, such as 61, 53, 41, 43, or 31, he or she can place a disc on any *prime* box.

Step 6: If a box is already occupied, a player cannot use it.

Step 7: The winner is the first player to cover four boxes horizontally, vertically, or diagonally from corner to corner.

How to Play *Factors*

Step 1: Choose the numbers with which you want to play. The numbers must be in consecutive order beginning with 1, and you must choose at least six numbers. Lay the numbers face up on a table. Put the other numbers aside.

Step 2: Pick a number from the table. You may only pick a number which has at least one of its factors on the table. You keep the number that you choose and give your opponent all its factors.

Step 3: Repeat Step 2 until you cannot pick any more numbers. Give your opponent the remaining numbers.

Step 4: Total your score and your opponent's score. Who won?

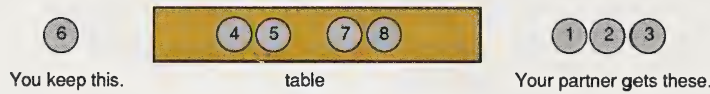
Step 5: Play another round. This time your opponent does the choosing.

Play the game several times. Try to discover a strategy so that you always win when it is your turn!

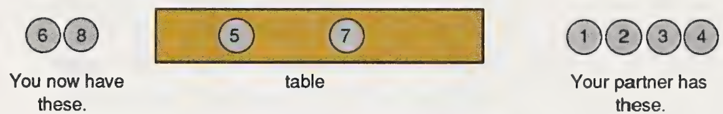
Example

Step 1: Say you choose eight numbers and lay the numbers on a table.

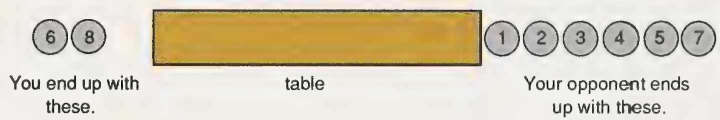
Step 2: Say you then pick the number 6. Since the factors of 6 are 3, 2, and 1, you give your opponent these numbers.



Step 3: You pick the number 8. You cannot pick 4, 5, or 7 because there are no factors of these numbers on the table. Since 4 is a factor of 8, your opponent gets 4.



You cannot pick any more numbers since there are no factors of 5 or 7 on the table. Your opponent gets 5 and 7.



Step 4: Your score is $6 + 8 = 14$. Your opponent's score is $1 + 2 + 3 + 4 + 5 + 7 = 22$. Your opponent is the winner!

Introductory Activities

Space for Your Work

Computer Alternative

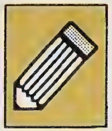
1. Play the game *Factors* on the *Number Munchers* disk (MECC).

2. Play the game *Primes* on the *Number Munchers* disk (MECC).

3. Play the game *Tax Collector* on the *Conquering Whole Numbers* disk (MECC).

4. To review prime numbers and prime factorization, do Lessons 14 and 15 of the *Numbers and Numeration* disk from the package *Computer Drill and Instruction: Mathematics, Level D* (SRA). Remember, if you need help press the SHIFT key and hold down the $\boxed{?}$ key.

Print Alternative



5. Play the game *Place Roll.*[1] You will need a pair of dice, and bingo chips or dried kernels of corn. The game board is in the appendix.

6. Play the game *Factors*. You will need to cut out the numbers in the appendix. Your goal is to get more points than your opponent.

7. List the factors of each of the following numbers.

   a. 36

   b. 42

   c. 30

Space for Your Work

[1] National Council of Teachers of Mathematics for excerpts from *The Arithmetic Teacher*, September, 1987, Reston, Virginia.

8. Give the prime factors of each of the following numbers.

   a. 36

   b. 42

   c. 30

9. Give the prime factorization of each of the following numbers.

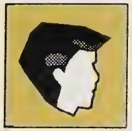
   a. 21

   b. 54

   c. 60

Space for Your Work

See your learning facilitator to check your answers and to receive further instructions.



Working Together

Now that you have reviewed the meaning of factors, you will learn about common factors and the greatest common factor.

Finding Common Factors

When a number is a factor of two or more numbers, it is a **common factor** of those numbers.

Example: What is the common factor of 6 and 15?

Solution

List all the factors of 6 and all the factors of 15.

6	
1	6
2	3
~~3~~	~~2~~

3 has already been mentioned.

15	
1	15
~~2~~	
3	5
~~4~~	
~~5~~	~~3~~

2 is not a factor.

4 is not a factor.

5 has already been mentioned.

The common factor of 6 and 15 is 3.

6	
1	6
2	3

15	
1	15
3	5

You can picture 6 like this.

2 x 3

You can picture 15 like this.

3 x 5

You can picture the common factor of 6 and 15 like this.

2 x 3 x 5

common factor

Some numbers have more than one common factor.

Example: What are two common factors of 24 and 30?

Solution

List all the factors of 24 and all the factors of 30.

24	
1	24
2	12
3	8
4	6
~~5~~	
~~6~~	~~4~~

5 is not a factor.

6 has already been mentioned.

30	
1	30
2	15
3	10
~~4~~	
5	6
~~6~~	~~5~~

4 is not a factor.

6 has already been mentioned.

Circle the common factors of 24 and 30.

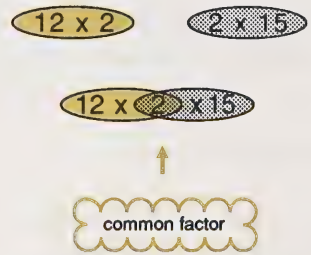
24	
1	24
2	12
3	8
4	6
~~5~~	
~~6~~	~~4~~

30	
1	30
2	15
3	10
~~4~~	
5	6
~~6~~	~~5~~

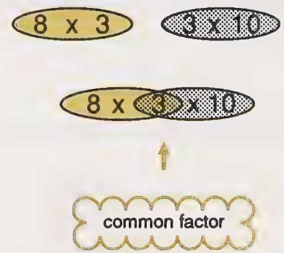
The common factors of 24 and 30 are 2, 3, and 6.

The common factors of 24 and 30 can be represented in the following way.

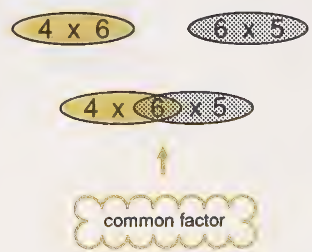
- 2 is a common factor of 24 and 30.



- 3 is a common factor of 24 and 30.



- 6 is a common factor of 24 and 30.



Finding the Greatest Common Factor

There are several methods you can use to find the greatest common factor of two or more numbers.

Method 1: Listing All the Factors

24	
1	24
2	12
3	8
4	6
~~5~~	
~~6~~	~~4~~

5 is not a factor.

6 has already been mentioned.

30	
1	30
2	15
3	10
~~4~~	
5	6
~~6~~	~~5~~

4 is not a factor.

6 has already been mentioned.

Circle the common factors of 24 and 30.

24	
1	24
2	12
3	8
4	6

30	
1	30
2	15
3	10
5	6

The **greatest common factor** (GCF) of 24 and 30 is 6.

Method 2: Dividing by the Common Prime Factors

$$\begin{array}{r|rr} 2 & 24 & 30 \\ \hline 3 & 12 & 15 \\ \hline & 4 & 5 \end{array}$$

These have no common prime factors.

The greatest common factor is the product of the common prime factors.

$$\begin{array}{r|rr} \boxed{2} & 24 & 30 \\ \hline \boxed{3} & 12 & 15 \\ \hline & 4 & 5 \end{array}$$

Multiply the factors in this column.

$$\text{GCF} = 2 \times 3$$
$$= 6$$

The greatest common factor of 24 and 30 is 6.

Method 3: Using Prime Factorization

Write the prime factorization for each number.

$$24 = 2 \times 2 \times 2 \times 3$$

$$30 = 2 \times 3 \times 5$$

Identify the common prime factors.

$$24 = \boxed{2} \times 2 \times 2 \times \boxed{3}$$

$$30 = \boxed{2} \times \boxed{3} \times 5$$

The common prime factors are 2 and 3.

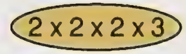
Multiply the common prime factors.

$$\text{GCF} = 2 \times 3$$

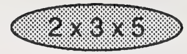
$$= 6$$

The greatest common factor of 24 and 30 is 6.

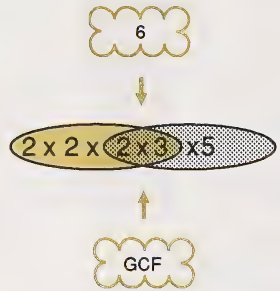
You can picture 24 this way.



You can picture 30 this way.



You can picture the GCF of 24 and 30 this way.



Applying Factors

How can factors be used in the everyday world?

Example 1

A physical education teacher can use factorization to decide which games the class can play so that all class members can participate. There are 32 class members. How many different ways can the children be grouped for games?

Solution

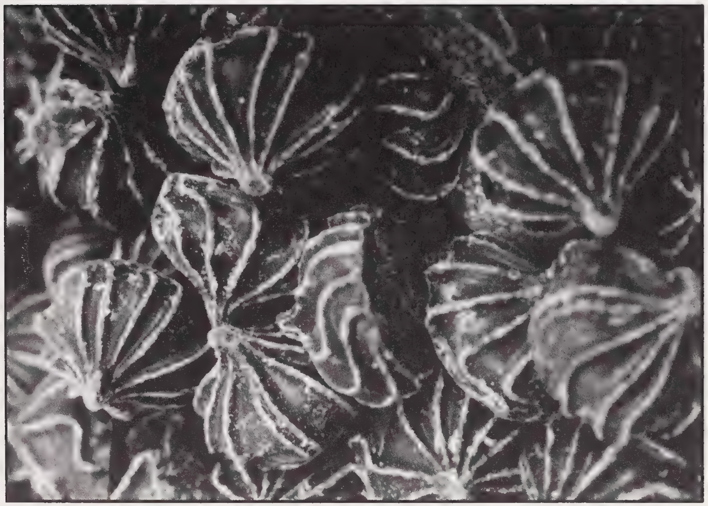
32	
1	32
2	16
4	8
16	2
32	1

Factors of 32 are 1, 2, 4, 8, 16, and 32.

The teacher can use games which require one group of 32 students, two groups of 16 students each, four groups of eight students each, eight groups of four students each, 16 groups of two children each, or 32 groups of one child each.

A grocer who wants to put several kinds of items into one package for a sale may need to know how to find the greatest common factor (GCF).

For example, imagine that a grocer wants to make up packages that contain two kinds of candy bars. Each package must have the same number of bars. Unfortunately, one type of candy comes in cases that contain 18 bars and the other comes in bags that contain 24. What is the largest number of packages that the grocer can make?



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Solution

$$\begin{array}{r|rr} 2 & 18 & 24 \\ \hline 3 & 9 & 12 \\ \hline & 3 & 4 \end{array}$$

$$\begin{aligned} \text{GCF} &= 2 \times 3 \\ &= 6 \end{aligned}$$

The grocer will end up with six packages of candy. Each package will contain three of one kind of candy bar and four of the other kind.

Practice Activities

Space for Your Work

1. List the factors of each of the following numbers.

   a. 54

   b. 81

   c. 63

2. a. List the common factors of 54, 81, and 63.

   b. What is the greatest common factor of 54, 81, and 63?

3. Find the greatest common factor for each of these groups of numbers.

   a. 6 and 12

   b. 30 and 45

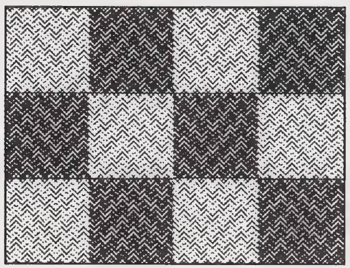
   c. 42 and 56

   d. 8, 16, and 20

   e. 24, 40, and 48

4. Helga bought some footballs for $117. Edie bought some of the same footballs for $78. What is the most that each football could have cost?

5. Julio is making a quilt out of squares of coloured material. His quilt is to be 126 cm wide and 198 cm long. What is the largest size of square that he can use?



See your learning facilitator to check your answers and to receive further instructions.

Space for Your Work

Extra Practice

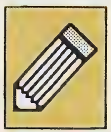
Space for Your Work

Computer Alternative

1. Do Lesson 16 of the disk *Numbers and Numeration* from the package *Computer Drill and Instruction: Mathematics, Level D* (SRA).

   Read the instructions included with the disk before using the program. If you need help, remember to hold down the SHIFT key and press the $\boxed{?}$ key.

Print Alternative



2. Find the GCF for each group of numbers.

   a. 24 and 36

   b. 12 and 18

   c. 30 and 75

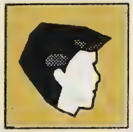
   d. 48, 64, and 80

   e. 500 and 300

3. Jennine, Julie, and Joyce bought the same kind of candies. Jennine bought 28¢ worth of candies, Julie bought 70¢ worth of candies, and Joyce bought 98¢ worth of candies. What is the most that each candy could have cost?

Space for Your Work

See your learning facilitator to check your answers and to receive further instructions.



Working Together

Here is another method for finding the greatest common factor of two numbers.

- Divide the larger number by the smaller number.
- Is the remainder zero? If no, divide the divisor by the remainder. If yes, the divisor is the GCF.

Example 1: What is the GCF of 20 and 30?

Divide 30 by 20.

$$\begin{array}{r} 1 \\ 20\overline{)30} \\ \underline{20} \\ 10 \end{array} \leftarrow \text{The remainder is not zero.}$$

Divide the divisor by the remainder.

$$\begin{array}{r} 2 \\ 10\overline{)20} \\ \underline{20} \\ 0 \end{array} \leftarrow \text{remainder of zero}$$

The last divisor is 10.

So, the GCF of 20 and 30 is 10.

Example 2: What is the GCF of 30 and 54?

Divide 54 by 30.

$$\begin{array}{r} 1 \\ 30\overline{)54} \\ \underline{30} \\ 24 \end{array} \leftarrow \text{The remainder is not zero.}$$

Divide the first divisor by the first remainder.

$$\begin{array}{r} 1 \\ 24\overline{)30} \\ \underline{24} \\ 6 \end{array} \leftarrow \text{The remainder is not zero.}$$

Divide the second divisor by the second remainder.

$$\begin{array}{r} 4 \\ 6\overline{)24} \\ \underline{24} \\ 0 \end{array} \leftarrow \text{remainder of zero}$$

The last divisor is 6.

So, the GCF of 30 and 54 is 6.

Two numbers with a greatest common factor of 1 are said to be **relatively prime**.

Example

- 10 and 21 are relatively prime.
  (The GCF of 10 and 21 is 1.)
- 10 and 15 are not relatively prime.
  (The GCF of 10 and 15 is 5.)

A **proper factor** of a number is less than the number itself.

Example: What are the proper factors of 10?

List the factors of 10.

10	
1	10
2	5
~~3~~	
~~4~~	
~~5~~	~~2~~

3 and 4 are not factors.

5 has already been mentioned.

The factors of 10 are 1, 2, 5, and 10.

The proper factors of 10 are 1, 2, and 5.

Concluding Activities

Space for Your Work

1. For each of the following give the smallest number that has exactly this number of factors.

   a. 3

   b. 4

   c. 5

   d. 6

   e. 7

   f. 8

   g. 9

   h. 10

2. a. Find an example in which the sum of the proper factors of a number is less than the number.

   b. Find an example in which the sum of the proper factors of a number is equal to the number.

   c. Find an example in which the sum of the proper factors of a number is more than the number.

3. Are the following pairs of numbers relatively prime? Answer **yes** or **no** and tell why.

   a. 87 and 53

   b. 21 and 36

   c. 45 and 38

   d. 21 and 93

   e. 74 and 26

Space for Your Work

See your learning facilitator to check your answers and to receive further instructions.

LEAST COMMON MULTIPLES

What Lies Ahead

In this section you will learn these skills.

- listing the multiples of a number
- finding the least common multiple (LCM)
- using the least common multiple (LCM) to solve problems

In this section you will use these words.

- multiple
- common multiple
- least common multiple (LCM)

Working Together

Do you remember what multiples are?

Multiples are products of a number and another whole number. They are usually larger than the original number.

As the name implies, multiples involve multiplying.

Example

The multiples of 5 have these values.

$$\underset{5 \times 1}{5}, \quad \underset{5 \times 2}{10}, \quad \underset{5 \times 3}{15}, \quad \underset{5 \times 4}{20}, \quad \ldots$$

Remember that multiplying is really repeated addition.

$$5, \quad \underset{5 + 5}{10}, \quad \underset{5 + 5 + 5}{15}, \quad \underset{5 + 5 + 5 + 5}{20}, \quad \ldots$$

You can use a calculator to help you find the multiples of a number.

There are several methods that can be used.

Method 1: Using Addition

Key Press	Display
9 + 9 =	18
+ 9 =	27
+ 9 =	36

Method 2: Using Automatic Constant

Key Press	Display
9 + 9 =	18
=	27
=	36

Method 3: Using Multiplication

Key Press	Display
1 x 9 =	9
2 x 9 =	18
3 x 9 =	27

Method 4: Using Automatic Constant

Key Press	Display
9 x 1 =	9
2 =	18
3 =	27

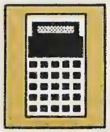
Note

Not all calculators have the automatic feature for addition or multiplication. Experiment with your calculator to see what it can do.

Introductory Activities

Space for Your Work

1. Use your calculator to find the first five multiples of each of these numbers.



   a. 9

   b. 12

   c. 23

Computer Alternative

2. If you want further practice, play *Multiples* on the *Number Munchers* disk (MECC).

See your learning facilitator to check your answers and to receive further instructions.

Working Together

Now that you have reviewed multiples you will learn about **common multiples** and the **least common multiple (LCM)**.

Example

What are three common multiples of 20 and 30? What is the least common multiple (LCM)?

Solution

Method 1

List the multiples of 20.

20, 40, 60, 80, 100, 120, 140, 160, 180, 200, ...

List the multiples of 30.

30, 60, 90, 120, 150, 180, ...

Circle the common multiples.

20, 40, 60, 80, 100, 120, 140, 160, 180, 200, ...

30, 60, 90, 120, 150, 180, ...

Three common multiples of 20 and 30 are 60, 120, and 180.

The least common multiple of 20 and 30 is 60.

Method 2

First divide by the common factors.

$$\begin{array}{r|rr} 2 & 20 & 30 \\ \hline 5 & 10 & 15 \\ \hline & 2 & 3 \end{array}$$

Then multiply the common factors by the remaining factors for each number. The factors to be multiplied form an *L*. To help you remember, think of Least Common Multiple.

$$\begin{array}{r|rr} 2 & 20 & 30 \\ \hline 5 & 10 & 15 \\ \hline & 2 & 3 \end{array}$$

$$\begin{aligned} \text{LCM} &= 2 \times 5 \times 2 \times 3 \\ &= 60 \end{aligned}$$

The least common multiple of 20 and 30 is 60.

Method 3

Here is a third way to calculate the LCM of 20 and 30.

Write the prime factorizations for each number.

$$20 = 2 \times 2 \times 5$$

$$30 = 2 \times 3 \times 5$$

Identify the common prime factors.

$$20 = \boxed{2} \times 2 \times \boxed{5}$$

$$30 = \boxed{2} \times 3 \times \boxed{5}$$

The common prime factors are 2 and 5.

The non-common prime factors are 2 and 3.

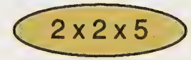
The LCM is the product of the common and non-common prime factors.

common prime factors | non-common prime factors

$$\text{LCM} = 2 \times 5 \times 2 \times 3$$

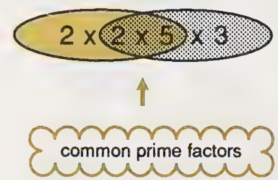
$$= 60$$

You can picture 20 like this.



You can picture 30 like this.

You can picture the LCM this way.



The LCM is the product of the common and non-common prime factors.

common prime factors | non-common prime factors

$$\text{LCM} = 2 \times 5 \times 2 \times 3$$

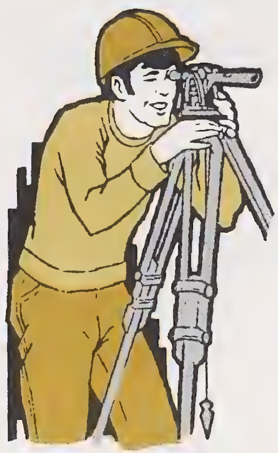
$$= 60$$

Applying Common Multiples

Factorization is very useful in the everyday world.

Example

A surveyor begins putting red and blue pegs into the ground at a particular tree. He puts a red peg every 30 m and a blue peg every 50 m. How far from the tree will he be when he puts in both a red peg and a blue peg?



Solution

Divide by the common factors.

$$\begin{array}{r|rr} 2 & 50 & 30 \\ \hline 5 & 25 & 15 \\ \hline & 5 & 3 \end{array}$$

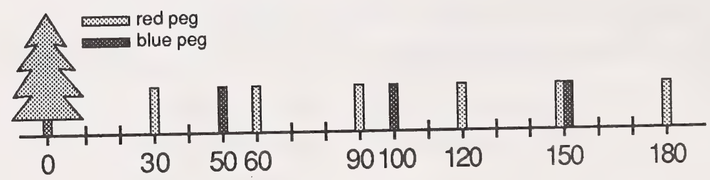
The LCM is the product of the common and non-common factors (Remember the *L*).

$$\text{LCM} = 2 \times 5 \times 5 \times 3$$

$$= 150$$

The surveyor will be 150 m from the tree when he puts in both a red and blue peg.

This diagram may help you visualize the situation.



Practice Activities

Space for Your Work

1. a. List the first 10 multiples of each of these numbers.

    - 6

    - 9

   b. Circle the common multiples of 6 and 9.

   c. Put a square around the least common multiple of 6 and 9.

2. Give the first three common multiples for each of the following pairs of numbers.

   a. 7 and 11

   b. 13 and 2

   c. 4 and 6

3. Find the LCM for each of these groups of numbers.

   a. 4, 6, and 10

   b. 3, 8, and 10

   c. 2, 3, 4, and 5

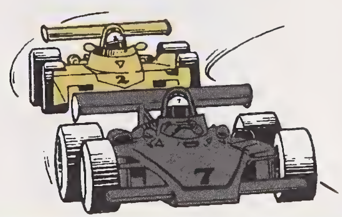
   d. 16 and 18

Space for Your Work

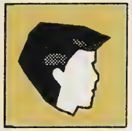
4. Camilla's race car takes nine minutes to circle the track while Kevin's goes around in 12 minutes.

   a. What is the least amount of time in which they could cross the finish line together?

   b. Who will have completed more laps? How many more?



See your learning facilitator to check your answers and to receive further instructions.



Working Together

You can use multiple boards to help you find the LCM.

Example 1: What is the LCM of 6 and 4?

Multiple board for 6

6	**12**	18	24	30	36	42

Multiple board for 4

4	8	**12**	16	20	24	28

The LCM of 4 and 6 is 12.

Here is a game to practise finding the LCM.

How to Play *The LCM Game*

Step 1: Roll the dice. The player with the highest total begins.

Step 2: In turn, each player throws the dice and by using the numbers on the top faces determines the least common multiple for those numbers.

Example:  The LCM is 12 since 12 is the smallest number that both 6 and 4 divide into evenly.

Step 3: The number found to be the LCM is covered with a disc by the player.

Step 4: If a player rolls a combination that is covered by the opponent, the player removes that disc and replaces it with a disc of his or her own colour.

Step 5: If the player rolls a combination that is covered by a disc of his or her own colour, the turn is lost.

Step 6: The game continues until one player is able to place four discs in a row horizontally, vertically, or diagonally.

Extra Practice

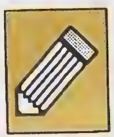
Space for Your Work

Computer Alternative

1. Do Lesson 17 of the disk *Numbers and Numeration* from the package *Computer Drill and Instruction: Mathematics, Level D* (SRA).

   Read the instructions included with the disk before you use the program. If you need help, remember to hold down the SHIFT key and press the ? key.

Print Alternative



2. Play *The LCM Game*[1]. You will need a partner, a pair of dice, and 20 discs (bingo chips or dried kernels of corn). The game board is in the appendix.

[1] National Council of Teachers of Mathematics for excerpts from the *The Arithmetic Teacher*, September, 1987, Reston, Virginia.

3. Find the LCM for each of the following pairs of numbers.

   a. 9 and 12

   b. 12 and 15

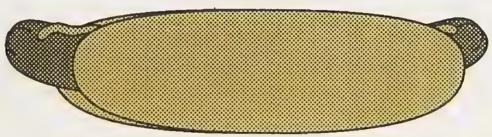
   c. 11 and 13

4. Find the LCM for the following groups of numbers.

   a. 14, 21, and 49

   b. 16, 18, and 24

5. You can buy wieners in packages of 10 and buns in packages of 12. If you want to have an equal number of wieners and buns, what is the lowest number of packages of each that you should buy?



Space for Your Work

Space for Your Work

6. Majeed replaces the plants in his restaurant to keep them looking fresh and nice. One type gets replaced every eight weeks. Another type gets replaced every 12 weeks.

   a. If Majeed begins on January 1, how many weeks will it be until both types of plants get replaced at the same time?

   b. How many times in the year will both types get replaced at the same time?

See your learning facilitator to check your answers and to receive further instructions.

Concluding Activities

Space for Your Work

1. a. Find the product of 18 and 24.

   b. Find the GCF and the LCM of 18 and 24.

   c. Find the product of the GCF and the LCM and compare it to your answer in Part a. of this question.

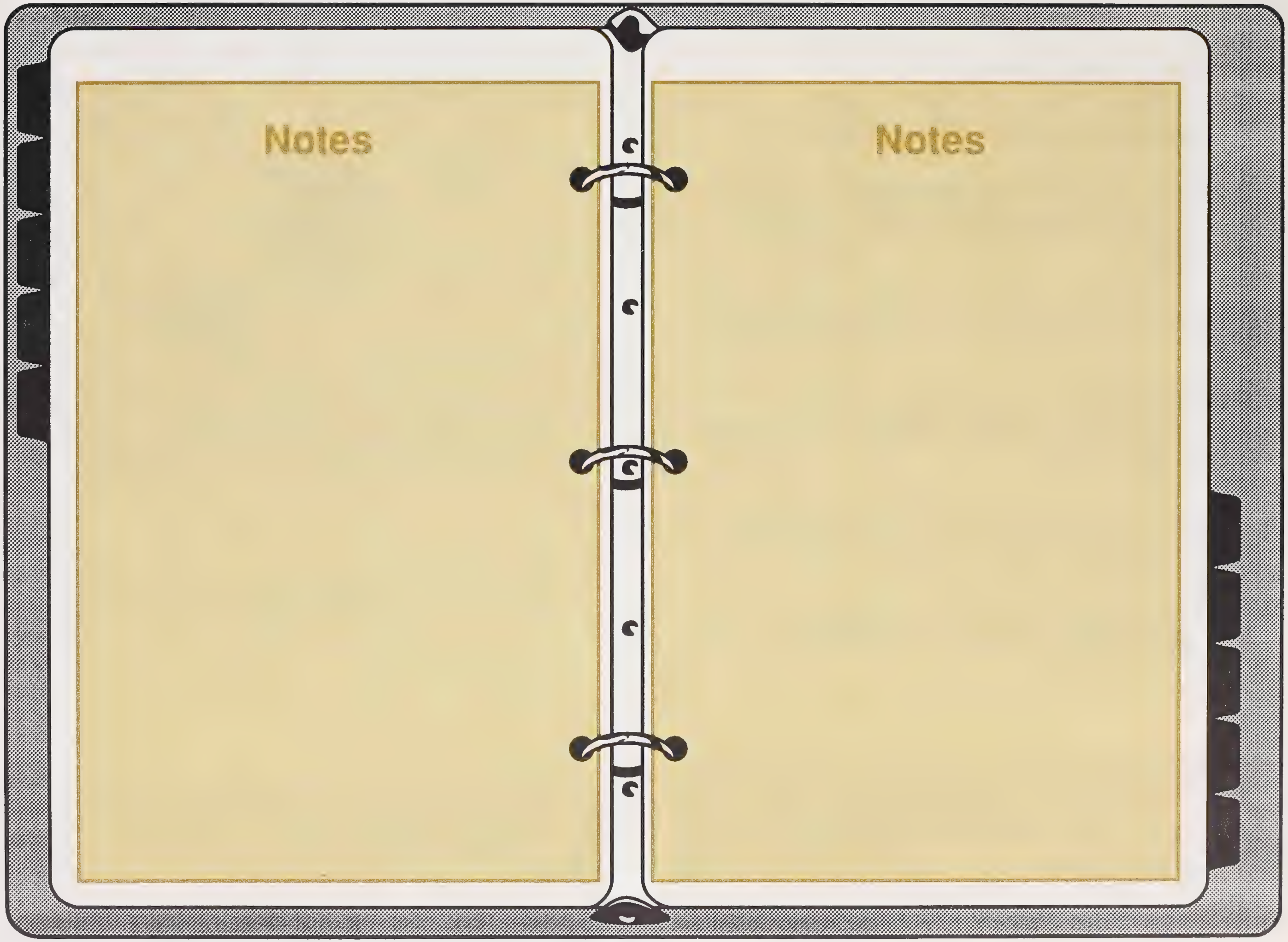
   d. Repeat the procedure for the numbers 20 and 56.

2. a. What statement can you make about the product of two numbers and the product of their GCF and LCM?

   b. How can you find the GCF of two numbers if you know the numbers and their LCM?

   c. Can you use the same idea to find the LCM if you know the numbers and their GCF?

See your learning facilitator to check your answers and to receive further instructions.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Notes

POWERS

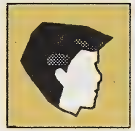
What Lies Ahead

In this section you will learn these skills.

- expressing a product of equal factors as a power
- calculating the value of powers (evaluating powers)

In this section you will use these words.

- exponent
- base
- power
- squared
- cubed
- standard form
- exponential form
- expanded form



Working Together

Do you know what a power is?

A **power** is a product of equal factors.

Example

The fourth power of 3 is 3^4.

$$3^4 = 3 \times 3 \times 3 \times 3$$
$$= 81$$

The power is made up of two parts – the **base** and the **exponent**.

3^4 ← The base is 3. It shows the number that is being multiplied by itself.

3^4 ← The exponent is 4. It shows how many times the base number is used as a factor.

Powers of 2 can also be demonstrated using base 10 blocks.

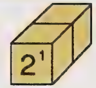
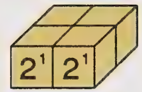
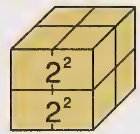
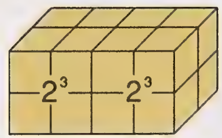
Look at the diagram to the right. Each power of 2 has twice as many blocks as the previous power of 2.

The first power of 2 is 2^1. It can be read as "2 exponent 1".

The second power of 2 is 2^2. It can be read as "2 exponent 2". You can also read 2^2 as "two **squared**".

The third power of 2 is 2^3. It can be read as "2 exponent 3". You can also read 2^3 as "two **cubed**".

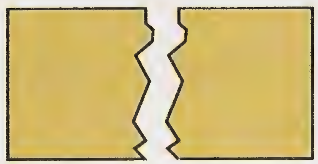
The fourth power of 2 is 2^4. It can be read as "2 exponent 4".

Power	Concrete Model	Meaning
2^1	 1 group of 2^1	$2^1 = 2$
2^2	 2 groups of 2^1	$2^2 = 2 \times 2^1$ $= 2 \times 2$ $= 4$
2^3	 2 groups of 2^2	$2^3 = 2 \times 2^2$ $= 2 \times (2 \times 2)$ $= 2 \times 4$ $= 8$
2^4	 2 groups of 2^3	$2^4 = 2 \times 2^3$ $= 2 \times (2 \times 2 \times 2)$ $= 2 \times 8$ $= 16$

Powers of 2

You can demonstrate the powers of 2 by repeatedly ripping a sheet of paper in two.

Step 1: Rip a large sheet of paper into halves.



$$2^1 = 2$$

Step 2: Next put the pieces on top of each other and tear them in half again. You will now have four pieces of paper.

$$\begin{aligned} 2^2 &= 2 \times 2 \\ &= 4 \end{aligned}$$

Step 3: Put all of the pieces on top of each other and tear them in half again. You will now have eight pieces of paper.

$$\begin{aligned} 2^3 &= 2 \times 2 \times 2 \\ &= 8 \end{aligned}$$

You can continue this process to find the other powers of 2.

$$\begin{aligned} 2^4 &= 2 \times 2 \times 2 \times 2 \\ &= 16 \end{aligned}$$

$$\begin{aligned} 2^5 &= 2 \times 2 \times 2 \times 2 \times 2 \\ &= 32 \end{aligned}$$

Numbers have many different but equivalent forms.

Example 1: Express 10^6 in **standard form**.

Solution

$$10^6 = 10 \times 10 \times 10 \times 10 \times 10 \times 10$$
$$= 1\,000\,000$$

Example 2: Express 625 in **exponential form**. This means express 625 as a power.

Solution

$$625 = 25 \times 25$$
$$= 25^2$$

$$625 = 5 \times 5 \times 5 \times 5$$
$$= 5^4$$

Example 3: Express 6290 in **expanded form**.

Solution

$$6290 = (6 \times 1000) + (2 \times 100) + (9 \times 10) + (0 \times 1)$$
$$= (6 \times 10^3) + (2 \times 10^2) + (9 \times 10^1) + (0 \times 1)$$

Example 4: Express $(7 \times 10^4) + (3 \times 10^2) + (9 \times 10^1) + (8 \times 1)$ in standard form.

Solution

$$(7 \times 10^4) + (3 \times 10^2) + (9 \times 10^1) + (8 \times 1) = 70\,398$$

Practice Activities

Space for Your Work

1. In each of the following expressions, tell which number is the exponent and which number is the base.

   a. 4^3

   b. 3^{10}

   c. 2^6

2. Express each of the following groups of factors as a power.

   a. $4 \times 4 \times 4 \times 4$

   b. $17 \times 17 \times 17$

   c. $121 \times 121 \times 121 \times 121 \times 121 \times 121 \times 121$

3. Write each number in standard form.

Space for Your Work

a. 3^5

b. 4^7

c. 11^4

d. 10^6

4. Complete the following table.

	Exponential Form	Base	Exponent	Meaning	Standard Form
a.	5^2				
b.	2^5				
c.		6	4		
d.				4×4	
e.	10^8				
f.		3	3		
g.				$7 \times 7 \times 7$	
h.		2	6		

5. Write each number in expanded form using powers.

   a. 983 765

   b. 1 083 975

6. Write each number in standard form.

   a. $(3 \times 10^5) + (4 \times 10^3) + (5 \times 10^2)$

   b. $(9 \times 10^6) + (5 \times 10^1) + (3 \times 1)$

Space for Your Work

See your learning facilitator to check your answers and to receive further instructions.

Extra Practice

Space for Your Work

Computer Alternative

1. Do Lessons 9, 10, and 11 of the disk *Numbers and Numeration* from the package *Computer Drill and Instruction: Mathematics, Level D* (SRA).

   Read the instructions included with the disk before using the program. If you need help, remember to hold down the SHIFT key and press the

   $\boxed{?}$ key.

Print Alternative

2. Express each expanded form as a power.

   a. $2 \times 2 \times 2 \times 2$

   b. $3 \times 3 \times 3 \times 3 \times 3$

   c. $5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5$

   d. $10 \times 10 \times 10 \times 10 \times 10 \times 10$

Space for Your Work

3. Express each power in standard form.

   a. 4^3

   b. 20^2

   c. 13^3

   d. 100^3

4. Express each number in expanded form using powers.

   a. 78 956

   b. 103 982

See your learning facilitator to check your answers and to receive further instructions.

Working Together

There are several ways to find the value of powers using calculators.

Example 1: Evaluate 8^2.

Solution

Method 1: Using Paper and Pencil

$$8 \times 8 = 64$$

Method 2: Using Automatic Constant

Key Press	Display
8 ×	8
=	64

Method 3: Using the x^2 Key

Key Press	Display
8	8
x^2	64

Example 2: Evaluate 8^5.

Solution

Method 1: Using Paper and Pencil

$$8 \times 8 \times 8 \times 8 \times 8 = 32\ 768$$

Method 2: Using Automatic Constant

Key Press	Display
8 ×	8
=	64
=	512
=	4 096
=	32 768

Method 3: Using the y^x Key

Key Press	Display
8	8
y^x 5 =	32 768

Division can be used to find a missing exponent.

Example: What is the missing exponent in $2^{\blacksquare} = 16$?

Solution

Method 1: Using Paper and Pencil

Keep dividing by 2.

See how many times 2 is a factor.

$16 \div 2 = 8$

$8 \div 2 = 4$

$4 \div 2 = 2$

$2 \div 2 = 1$

Since 2 is used as a factor four times, the exponent is 4.

So, $2^4 = 16$.

Method 2: Using Automatic Constant

Key Press	Display
$\boxed{1}\boxed{6}\boxed{\div}\boxed{2}\boxed{=}$	8
$\boxed{=}$	4
$\boxed{=}$	2
$\boxed{=}$	1

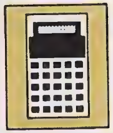
Since the $\boxed{=}$ key is pressed four times to reach 1 in the display, the exponent is 4.

So, $2^4 = 16$.

Concluding Activities

Space for Your Work

1. Evaluate each of these powers using a calculator.



a. 5^6

b. 3^3

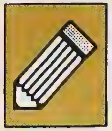
c. 25^4

d. 186^2

e. 9^7

f. 23^4

2. Complete each power by supplying an exponent.



a. $128 = 2^{\square}$

b. $32\,768 = 8^{\square}$

c. $279\,936 = 6^{\square}$

d. $3\,375 = 15^{\square}$

3. a. The standard form of any power with base 5, such as 5^3, will have what number as its final digit?

b. The standard form of any power with base 6, such as 6^3, will have what number as its final digit?

c. Can you find any other numbers whose powers all have the same last digit when written in standard form? What are they?

Space for Your Work

Space for Your Work

4. Each number has a repeating pattern of last digits for its powers. For example, all powers of 2 end in 2, 4, 6, or 8.

$2^1 = \mathbf{2}$	$2^5 = 32$
$2^2 = \mathbf{4}$	$2^6 = 64$
$2^3 = \mathbf{8}$	$2^7 = 128$
$2^4 = 16$	$2^8 = 256$

So, the last digit repeating pattern for base 2 is

2, 4, 8, 6.

Complete the last digit repeating pattern for the following numbers.

a. 3

b. 4

c. 7

d. 8

e. 9

5. Use the patterns you discovered in Question 4 to help you determine the last digits for these powers when expressed in standard form.

   a. 5^{11}

   b. 4^{21}

   c. 3^{9}

   d. 2^{49}

Space for Your Work

Space for Your Work

6. Find the squares for each of these numbers.

   a. 15

   b. 25

   c. 35

   d. 45

7. a. What pattern did you notice in Question 6?

   b. Use the pattern from Question 6 to find 55^2 and 65^2.

See your learning facilitator to check your answers and to receive further instructions.

SCIENTIFIC NOTATION

What Lies Ahead

In this section you will learn these skills.

- writing numbers in scientific notation
- writing scientific notation in standard form

In this section you will use these words.

- scientific notation
- standard form
- power of 10

Working Together

You have learned that numbers can have many different but equivalent forms. Numbers can be expressed in standard form, expanded form, and also in **scientific notation**.

In scientific notation a whole number is written as a product of a number between 1 and 10 and a **power of 10**.

$965\ 000\ 000$	standard form
$(9 \times 10^8) + (6 \times 10^7) + (5 \times 10^6)$	expanded form
$\underbrace{9.65}_{\text{number between 1 and 10}} \times \underbrace{10^8}_{\text{a power of 10}}$	scientific notation

The Introductory Activities will deal with powers of 10.

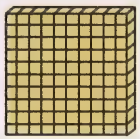
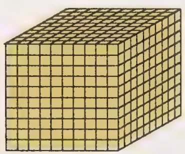
Introductory Activities

Space for Your Work

1. First review what the concrete models of the powers of 10 look like. Then answer the following questions.

   a. What would 10^4 look like?

   b. What would 10^5 look like?

Power	Concrete Model	Value and Standard Form
10^1	 10 groups of 10	$10^1 = 10 \times 1$ $= 10$
10^2	 10 groups of 10^1	$10^2 = 10 \times 10^1$ $= 10 \times 10$ $= 100$
10^3	 10 groups of 10^2	$10^3 = 10 \times 10^2$ $= 10 \times 10 \times 10$ $= 1\ 000$

2. Evaluate each of these powers. Part a. is done as an example.

   a. 10^4

   b. 10^5

   c. 10^8

   d. 10^{10}

3. Use the results from Question 2 to complete the following statements.

   a. When the exponent is 4 and the base is 10, there will be _____ zeros in the answer.

   b. When the exponent is 5 and the base is 10, there will be _____ zeros in the answer.

   c. When the exponent is 8 and the base is 10, there will be _____ zeros in the answer.

   d. When the exponent is 10 and the base is 10, there will be _____ zeros in the answer.

Space for Your Work

2. a. $10^4 = 10 \times 10 \times 10 \times 10 = 10\,000$

4. Complete each power by supplying an exponent. Part a. is done as an example.

   a. $10\,000 = 10^{▒}$

   b. $1\,000\,000 = 10^{▒}$

   c. $1\,000\,000\,000 = 10^{▒}$

   d. $100\,000 = 10^{▒}$

5. Use the results from Question 4 to complete these statements.

   a. 10 000 has four zeros and the exponent in the power of 10 is _____ .

   b. 1 000 000 has six zeros and the exponent in the power of 10 is _____ .

   c. 100 000 000 has eight zeros and the exponent in the power of 10 is _____ .

   d. 100 000 has five zeros and the exponent in the power of 10 is _____ .

Space for Your Work

4. a. $10\,000 \div 10 = 1\,000$

   $1\,000 \div 10 = 100$

   $100 \div 10 = 10$

   $10 \div 10 = 1$

   Using the division method, four steps were needed to reach 1.

   So, $10\,000 = 10^4$.

6. Change each of these to standard form. Part a. is done as an example.

   a. 1.575×10^5

   b. 1.325×10^3

   c. 4.76×10^4

   d. 3.82×10^2

7. Use the results from Question 6 to complete these statements.

   a. When you multiply a number by 10^5, the decimal point moves ____ places to the right.

   b. When you multiply a number by 10^3, the decimal point moves ____ places to the right.

   c. When you multiply a number by 10^4, the decimal point moves ____ places to the right.

   d. When you multiply a number by 10^2, the decimal point moves ____ places to the right.

See your learning facilitator to check your answers and to receive further instructions.

Space for Your Work

6. a. $1.575 \times 10^5 = 1.575 \times 100\ 000$

$= 157\ 500$

Working Together

Why Scientific Notation was Developed

Scientific notation was developed by scientists to express very large numbers in a more compact way.

In scientific notation a whole number is written as a number between 1 and 10 times a power of 10.

This makes large numbers easier to read, compare, and remember.

Example 1

52 cards can be arranged in approximately 81 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 different ways.

This example would look like this when written in scientific notation.

52 cards can be arranged in approximately 8.1×10^{67} different ways.

Example 2

Each year about 4 100 000 000 000 matches are produced in the world.

The same statement would be written in the following way when using scientific notation.

Each year about 4.1×10^{12} matches are produced in the world.

Practice Activities

Space for Your Work

1. Which of the following numbers are written in scientific notation?

   a. 6.7×10^{23}

   b. 7 000

   c. 0.4×10^{2}

   d. 9×10^{1}

   e. 13×10^{29}

   f. 1.89×5^{10}

2. Complete each scientific notation by supplying an exponent.

   a. $312.4 = 3.124 \times 10^{\square}$

   b. $940\,000 = 9.4 \times 10^{\square}$

   c. $10.43 = 1.043 \times 10^{\square}$

   d. $6\,000\,600 = 6.0006 \times 10^{\square}$

3. Complete the scientific notation for each of the following by writing the correct decimal number in the box.

   a. 40 962 000 = $\boxed{\quad\quad} \times 10^7$

   b. 1 002 = $\boxed{\quad\quad} \times 10^3$

   c. 980 000 000 = $\boxed{\quad\quad} \times 10^8$

   d. 55 555 = $\boxed{\quad\quad} \times 10^4$

4. Express each of these numbers in scientific notation.

   a. 4 000

   b. 312 000

   c. 9 002 000

   d. 63 500 000 000

5. Express each of the following in scientific notation.

   a. The mass of Earth is 5 984 000 000 000 000 000 t.

   b. The number of salt water molecules on Earth is 120 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000.

   c. The circumference of Jupiter is about 434 000 km.

Space for Your Work

6. Express each of the following in standard form.

a. The distance from the sun to Earth is about 1.5×10^{8} km.

b. The distance from the sun to Jupiter is about 7.8×10^{9} km.

c. The number of cells in a bacterial culture is 4.03×10^{12} cells.

See your learning facilitator to check your answers and to receive further instructions.

Space for Your Work

Extra Practice

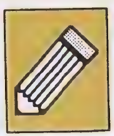
Space for Your Work

Computer Alternative

1. Do Lessons 11 and 12 on the disk *Numbers and Numeration* from the package *Computer Drill and Instruction: Mathematics Level D* (SRA).

   Read the instructions in the folder with the disk before using the program. If you need help, remember to hold down the SHIFT key and press the $\boxed{?}$ key.

Print Alternative



2. Complete each of these expressions.

   a. $23.3 = 2.33 \times 10^{\square}$

   b. $10\,090 = \square \times 10^4$

   c. $400\,090 = \square \times 10^5$

   d. $290 = 2.9 \times 10^{\square}$

3. Express each of the following in scientific notation.

   a. 872 000

   b. 891 700 000

   c. 532 800

   d. 2 004 000 000

4. Change each of the following to standard form.

   a. 7.241×10^5

   b. 1.4×10^7

   c. 3.974×10^8

   d. 5.055×10^2

Space for Your Work

Space for Your Work

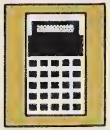
5. Mr. Rajoo's heart beats 70 times in one minute. Calculate the number of times his heart beats over a lifespan of 75 years. Express your answer in scientific notation.

6. If light travels at the speed of 300 000 kilometres per second, how far does light travel in one year? Express your answer in scientific notation.

See your learning facilitator to check your answers and to receive further instructions.

Concluding Activities

Space for Your Work



1. a. What is the greatest number, in standard form, that can be displayed on most simple calculators?

   b. What happens when you try to enter 1 673 195 054 into a simple calculator?

   c. What happens when you multiply 87 000 $\times$ 5 670 on a simple calculator?

   d. What happens when you add 972 167 and 8 616 790 on a simple calculator?

Space for Your Work

2. Scientific calculators use scientific notation to display larger numbers. However, these calculators do not display the multiplication sign and the power of 10. Instead, a space is left between the number and the exponent.

   Example: When written in scientific notation, a number looks like this.

   $$5.138 \times 10^{8}$$

   When displayed in scientific notation on your calculator, the same number looks like this.

   5.138 8

   What do the following displays mean?

   a. 4.2719 6

   b. 3.2719 15

See your learning facilitator to check your answers and to receive further instructions.

SUMMARY

What Lies Ahead

In this section you will review these skills.

- finding the factors of a number
- finding the common factors and the GCF of two or more numbers
- finding the multiples of a number
- finding the common multiples and the LCM of two or more numbers
- expressing numbers in standard form as powers
- expressing powers as numbers in standard form
- expressing numbers in standard form as numbers in expanded form
- expressing numbers in expanded form as numbers in standard form
- expressing numbers in scientific notation

Working Together

At this point it is a good idea to review the skills you have learned in sections 2 to 6.

Turn to Section 2 and review the pretest. Correct any errors you may have made. You may be pleasantly surprised to discover how much you have learned.

PART THREE

Part Three deals with operations with integers. You will be finding sums, differences, products, and quotients using mental computation, paper and pencil, and a calculator. You will also use the rules for order of operations to perform a series of operations.

What Lies Ahead

This section will pretest the following skills.

- adding integers
- subtracting integers
- multiplying integers
- dividing integers
- performing a series of operations

Working Together

The pretest will help you and your learning facilitator to determine your strengths and weaknesses.

Pretest

Space for Your Work

1. Write an integer which represents

   a. a pay raise of $300.

   b. a cheque written on your bank account for $21.

   c. 6 700 metres below sea level.

   d. a golf score of six over par.

   e. 14 degrees below the freezing point.

2. Write the opposite integer for each of the following.

   a. -46

   b. 198

   c. -230

   d. $+99$

3. Use > or < to show the relationship between the integers given in each of these cases.

   a. 49 ● 41

   b. − 49 ● − 41

   c. 7 ● − 7

   d. − 2 ● 1

   e. − 31 ● − 30

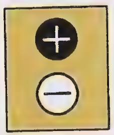
4. Arrange each group of integers in increasing order.

   a. − 2, + 14, − 20, − 14, + 7, 0

   b. + 9, + 8, − 36, − 20, − 1, + 100, − 1 000

Space for Your Work

5. Use counters and a container to find these sums and then complete each addition sentence.



a. $(+9) + (+2)$

b. $(-6) + (+3)$

c. $(+4) + (-6)$

d. $(-4) + (-3)$

Space for Your Work

6. Add these integers mentally.

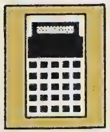
a. $(+12) + (-3)$

b. $(-4) + (-8)$

c. $(+2) + (+3)$

d. $(-12) + (+8)$

7. Use a calculator to answer these problems.

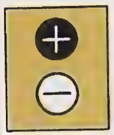


Space for Your Work

a. One day this winter the temperature was $-13°C$. It dropped another 10 degrees during that day and a further six degrees overnight. What was the coldest temperature recorded on this particular day?

b. The next day a chinook raised the temperature by 18 degrees. What was the resulting temperature?

8. Use counters and a container to find each of these differences.



a. $(+4) - (+2)$

b. $(+6) - (-2)$

c. $(-5) - (+3)$

d. $(-7) - (-2)$

9. Compute each of the following mentally or use paper and pencil.

a. $(+20) - (+18)$

b. $(+43) - (-21)$

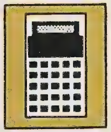
c. $(-15) - (+30)$

d. $(-11) - (-29)$

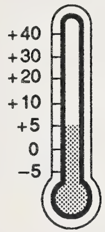
Space for Your Work

10. Use a calcultor to answer these problems.

Space for Your Work

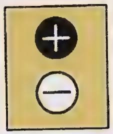


a. Yesterday the temperature in Barrhead was – 12°C. Today it is 5°C. How many degrees has the temperature risen?



b. The highest spot in North America is Mt. McKinley in Alaska at 6 194 metres. The lowest is in Death Valley at – 86 metres. What is the difference in the heights of these two places?

11. Use counters and a container to find each of these products.



a. $(+3) \times (+2)$

b. $(+4) \times (-2)$

c. $(-2) \times (+3)$

d. $(-3) \times (-4)$

12. Find each of these products mentally.

a. $(+4) \times (+5)$

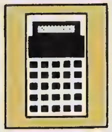
b. $(+7) \times (-3)$

c. $(-8) \times (+4)$

d. $(-10) \times (-6)$

Space for Your Work

13. Use your calculator to find each of these products.



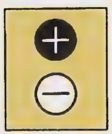
a. $(+12) \times (-16) = $ ▒

b. $(-48) \times (+34) = $ ▒

c. $(-62) \times (-49) = $ ▒

d. $(+19) \times (+87) = $ ▒

14. Use counters and a container to find each of these quotients.



a. $(+6) \div (+2)$

b. $(-6) \div (+2)$

c. $(+6) \div (-3)$

d. $(-10) \div (-2)$

Space for Your Work

Space for Your Work

15. Find each of these quotients mentally or by using paper and pencil methods.

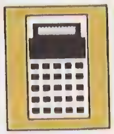
a. $(+69) \div (+3) = $ ▩

b. $(+74) \div (-37) = $ ▩

c. $(-24) \div (+3) = $ ▩

d. $(-50) \div (-10) = $ ▩

16. Use your calculator to find each of these quotients.



a. $(+117) \div (+13)$

b. $(-475) \div (+19)$

c. $(+255) \div (-15)$

d. $(-289) \div (-17)$

17. Use the rules for order of operations to evaluate each of the following.

Space for Your Work

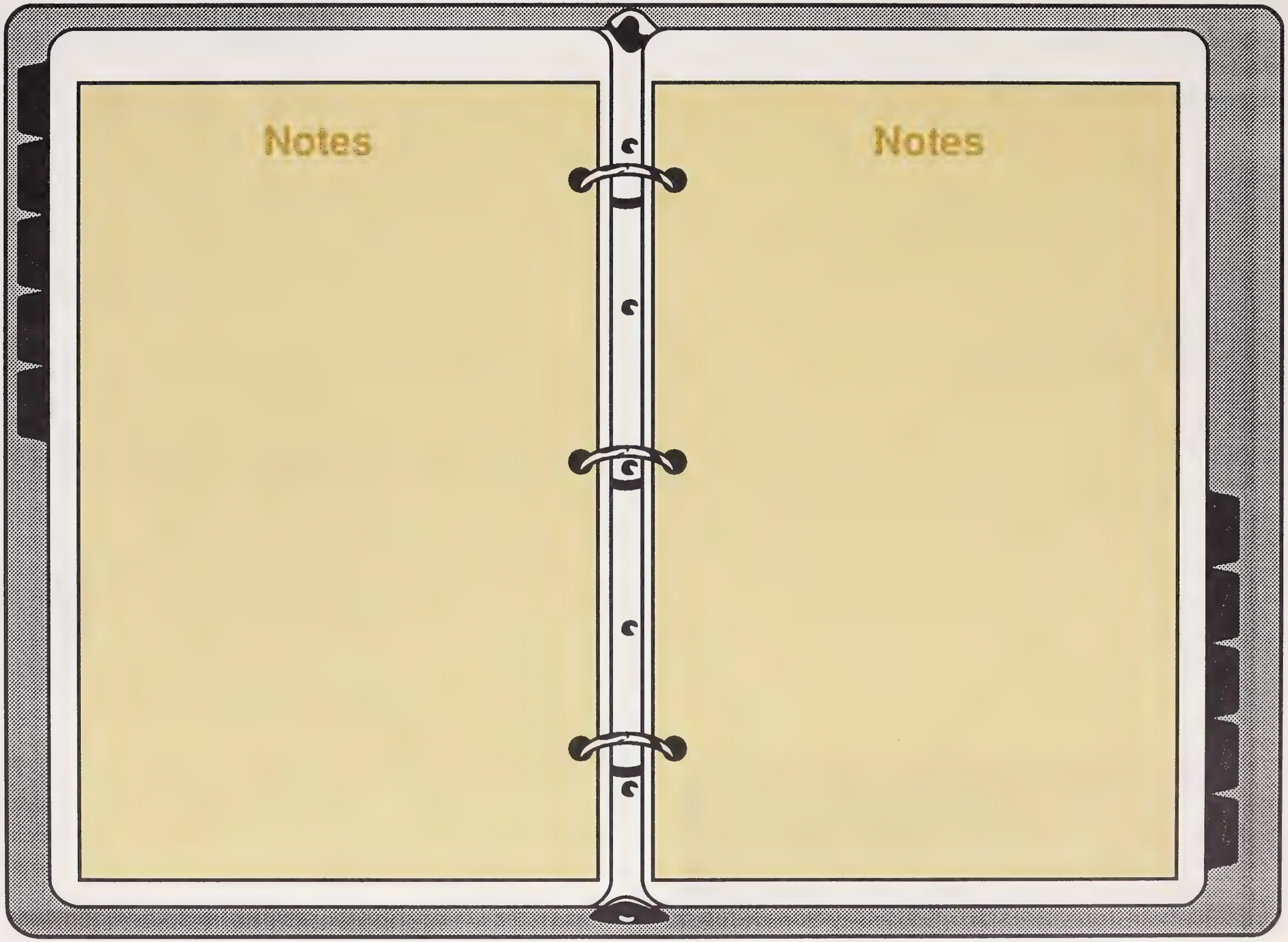
a. $(-6) + (-2) \times (-4)$

b. $(-8) \div (-4) \times (-3) - (-5)$

c. $\dfrac{(-4) + (+2) - (-3)}{(+8) \times (-3) \div (-24)}$

d. $[(+16) - (+2) \times (-8)] \div [(-3) \times (+4) \div (-6)]$

See your learning facilitator to check your answers and to receive further instructions.


Notes
Notes

INTEGERS

What Lies Ahead

In this section you will learn these skills.

- interpreting integers
- reading and writing integers
- comparing integers
- ordering integers

In this section you will use these words.

- integer
- positive integer
- negative integer
- opposite integers
- number line

Working Together

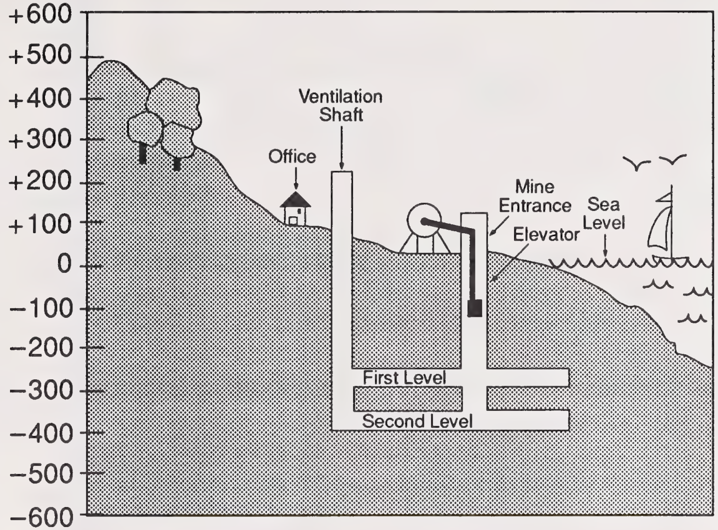
In everyday life quantities are often measured in two opposite directions from a starting point. Zero is used to indicate the starting point. **Integers** show the direction and distance from the starting point. Here are some examples.

- degrees below freezing; degrees above freezing
- years BC; years AD
- degrees of latitude south of the equator; degrees of latitude north of the equator
- degrees of longitude west of Greenwich, England; degrees of longitude east of Greenwich, England
- height above sea level; height below sea level
- golf score above par; golf score under par

Integers are made up of two parts – a sign and a number. The sign indicates the direction from the starting point (zero) and the number indicates the number of units from zero.

Example

Consider this drawing.



Location with Respect to Sea Level (m)

Integers can be used to describe the location of objects.

- Sea level, the starting point, is at 0 m.
- The metres above sea level are positive.
- The metres below sea level are negative.

You can use the following integers to describe the location of objects in this drawing.

- The boat is at sea level, 0 m.
- The office is 100 m above sea level, + 100 m.
- The elevator is 100 m below sea level, – 100 m.

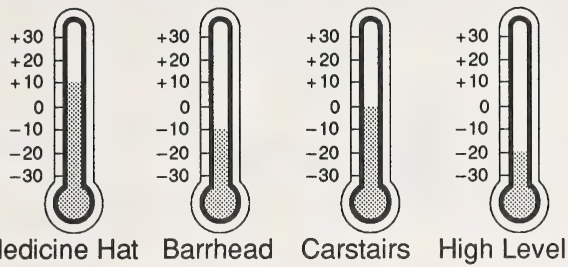
The office and the elevator are approximately the same distance from sea level, but in opposite directions. So, +100 and −100 are **opposite integers**.

Comparing and Ordering Integers

You learned to order and compare whole numbers in earlier grades. You will need to use that skill when comparing and ordering integers.

Example

The thermometers show the temperatures in four different places in Alberta on a winter day. Compare the temperatures using the following diagrams. Then arrange the communities in decreasing order of temperature.



Solution

- The temperature at Medicine Hat is greater than the temperature at Carstairs.

$$+10 > 0$$

- The temperature at Carstairs is greater than the temperature at Barrhead.

$$0 > -10$$

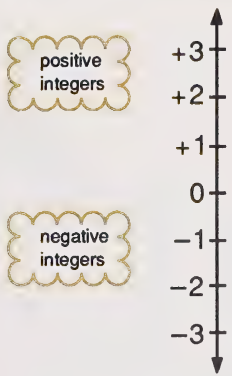
- The temperature at Barrhead is greater than the temperature at High Level.

$$-10^\circ > -20^\circ$$

The communities arranged in decreasing order of temperature are Medicine Hat, Carstairs, Barrhead, and High Level.

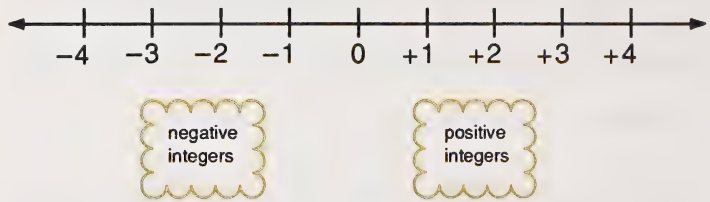
Using Number Lines to Compare and Order Numbers

A vertical number line can be used to compare and order **positive integers** and **negative integers**.



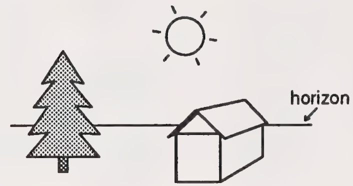
For two integers on a vertical number line, the integer higher up on the number line has the greater value.

A horizontal number line can also be used to compare and order integers.



For two integers on a horizontal number line, the integer farther to the right has the greater value.

To remember which is horizontal, think of the horizon.



Example 1

Compare the following pairs of integers.

- $+5$ and -10
- 0 and -5
- -10 and -5

Solution

Use a number line.

- Because $+5$ is to the right of -10, $+5 > -10$.
- Because 0 is to the right of -5, $0 > -5$.
- Because -10 is to the left of -5, $-10 < -5$.

Example 2

Order $0, -2, +3, -3$, and $+1$ from smallest to largest.

Solution

Use a number line.

From smallest to largest the integers are $-3, -2, 0, +1$, and $+3$.

Practice Activities

Space for Your Work

1. Write the positive or negative integer that describes each of these situations.

   a. two floors below ground level

   b. 6 km above the ground

   c. five bonus points

   d. 12 m below sea level

   e. nine degrees above the freezing point

2. Write the opposite integer for each integer given.

   a. $+3$

   b. -13

   c. 43

   d. $-1\ 001$

   e. -1

3. Find the pair of opposite integers in each group.

a. $+17, +70, -7, -700, -17$

b. $403, -430, +304, -403, -433$

c. $3, 33, -333, -3, -3\,333$

4. In a golf game, -2 means two strokes under par. What does each of the following integers mean when expressed as a golf score?

a. -6

b. 2

c. 0

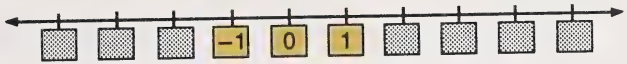
d. 14

e. -8

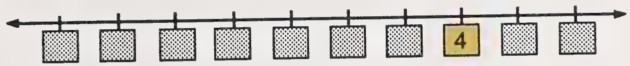
Space for Your Work

5. Complete each of the following number lines.

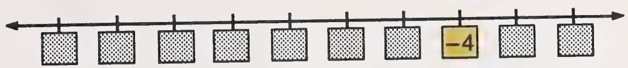
a.



b.



c.



Space for Your Work

6. Use > or < to show the relationship between each pair of integers.

a. 6 ● 16

b. – 5 ● – 3

c. – 2 ● – 20

d. – 12 ● 1

e. 4 ● + 5

7. Arrange each group of integers in order from least to greatest.

   a. $-4, 8, -3, 12, 0$

   b. $+6, 0, -4, -8, +11, +4, -10, -5$

   c. $-1, -3, -6, +8, +13, +11, 0, -15$

Space for Your Work

See your learning facilitator to check your answers and to receive further instructions.

Extra Practice

Space for Your Work

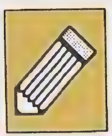
Computer Alternative

1. Do Lessons 11 and 12 on the disk *Pre-Algebra D* from the package *Computer Drill and Instruction: Mathematics, Level D* (SRA).

   Read the instructions included with the disk before using the program. If you need help, remember to hold down the SHIFT key and press the

   ? key.

Print Alternative



2. Use integers to describe each of these situations.

   a. a raise of 7%

   b. a loss of $40

   c. a gain of 30 yards

   d. four strokes below par

e. 40 degrees below zero Celsius

f. bank account withdrawal of $50

g. 120 m above sea level

h. a profit of $10 609

3. Order each of these groups of integers from least to greatest.

a. $-1, -2, -6, +8, +10, 0, +1$

b. $33, 13, 3, -3, -13, -33$

c. $0, 2, 4, -2, -63, 129, -10\,468$

Space for Your Work

See your learning facilitator to check your answers and to receive further instructions.

Working Together

The absolute value of an integer describes its distance from zero without indicating the direction.

Example 1: What is the absolute value of + 3?

The absolute value of + 3 is 3. It is written this way.

$|+3| = 3$

Example 2: What is the absolute value of − 3?

The absolute value of − 3 is 3. It is written this way.

$|-3| = 3$

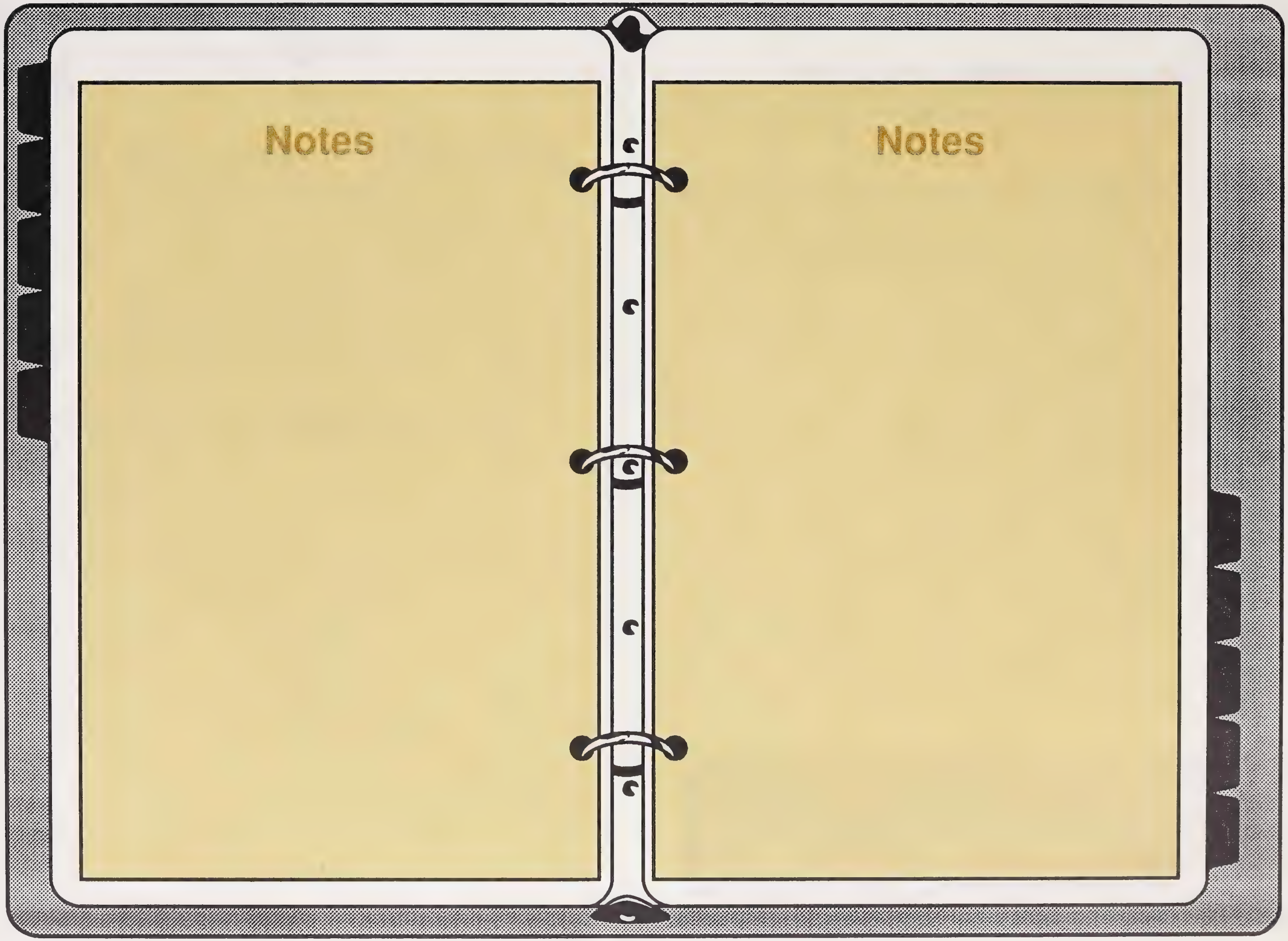
Concluding Activities

Space for Your Work

Give the absolute value of the following integers.

1. $+5$
2. -5
3. $+7$
4. -7
5. 0

✓ See your learning facilitator to check your answers and to receive further instructions.


Notes
Notes

ADDING INTEGERS

What Lies Ahead

In this section you will review this skill.

- adding integers using objects

You will also learn these skills.

- adding integers without using objects
- adding integers using a calculator

Working Together

In the last section you learned to compare and order integers and to relate integers to everyday events.

In the next four sections you will learn how to perform operations and solve problems with integers. You will learn to compute both with and without learning aids (manipulatives). You will also learn to use your calculator to do these operations.

You will begin this section by adding integers.

Using Counters to Model Integers

To model integers with counters, think of a counter as either having a positive electrical charge or a negative electrical charge.

How do you model + 3?

You can use ⊕⊕⊕ to model + 3.

How do you model – 4?

You can use ⊖⊖⊖⊖ to model – 4.

How do you model zero?

Since a positive charge and a negative charge are opposites, combining a positive counter and a negative counter will balance the charges and give no charge (zero). You can use any number of pairs to represent zero as long as there are the same number of positive counters and negative counters.

You can use ⊕⊖ to model zero.

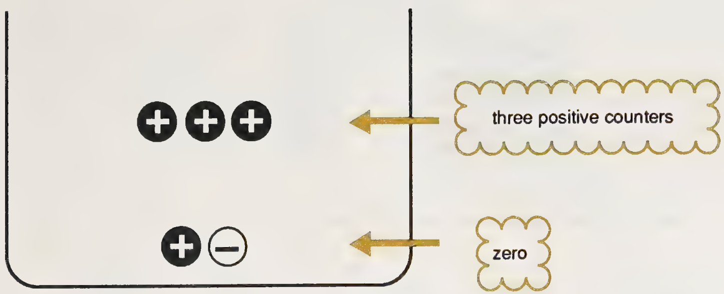
You can also use ⊕⊕⊕⊕ ⊖⊖⊖⊖ to model zero.

Two numbers whose sum is zero are called **additive inverses**.

- + 1 and – 1 are additive inverses.
- + 4 and – 4 are additive inverses.

Example 1

What is the charge in this container?

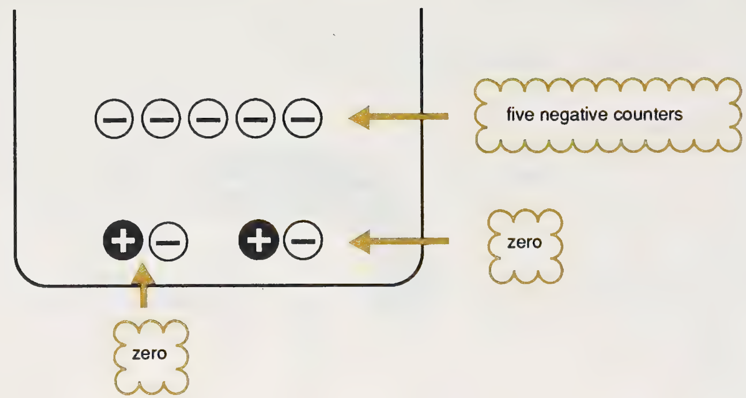


A positive counter and a negative counter neutralize each other. There is a surplus of three positive counters.

So, the charge is + 3.

Example 2

What is the charge in this container?



Two positive counters and two negative counters neutralize each other. There is a surplus of five negative counters.

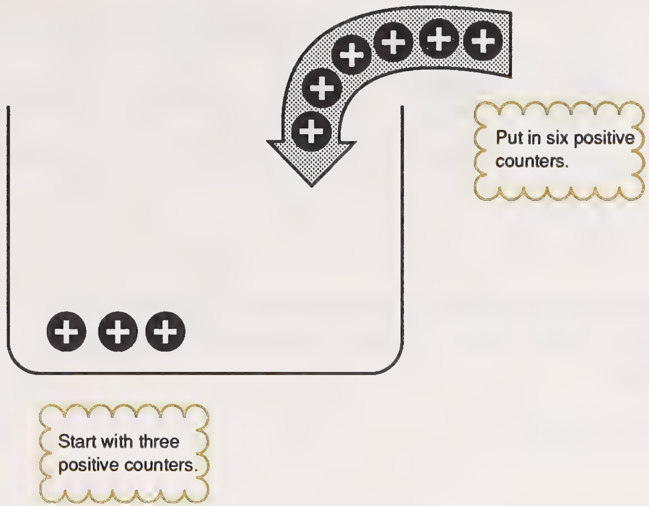
So, the charge is – 5.

Adding Integers with Counters

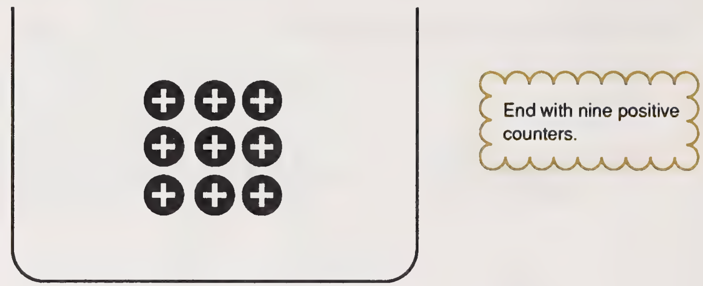
You can think of adding integers as putting counters into a container.

Example 1: What is $(+3) + (+6)$?

Solution



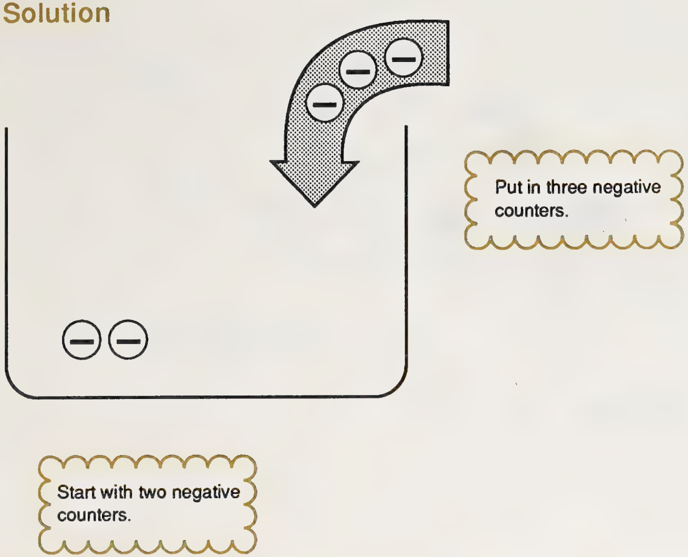
The result is nine positive counters in the container.



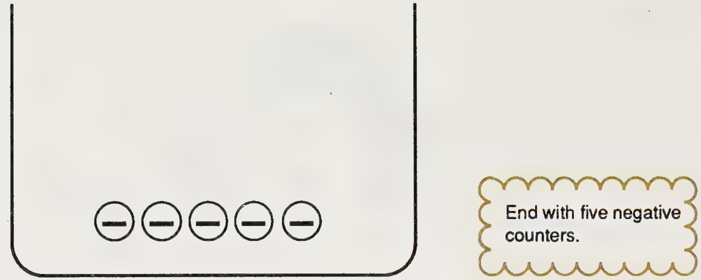
So, $(+3) + (+6) = +9$.

Example 2: What is $(-2) + (-3)$?

Solution



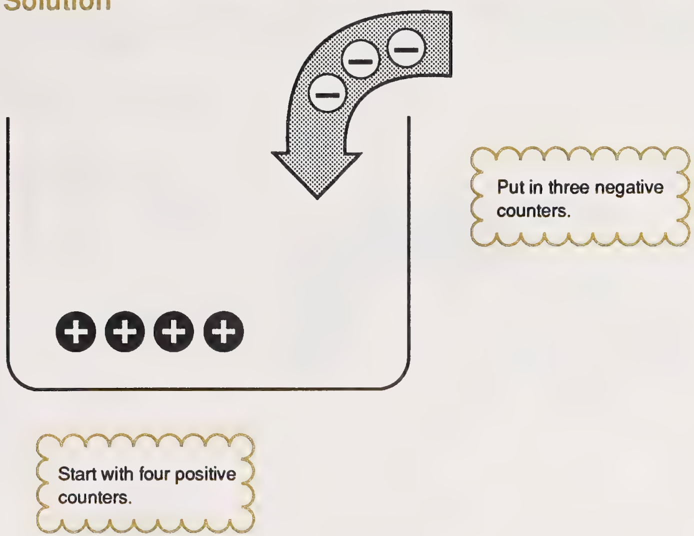
The result is five negative counters in the container.



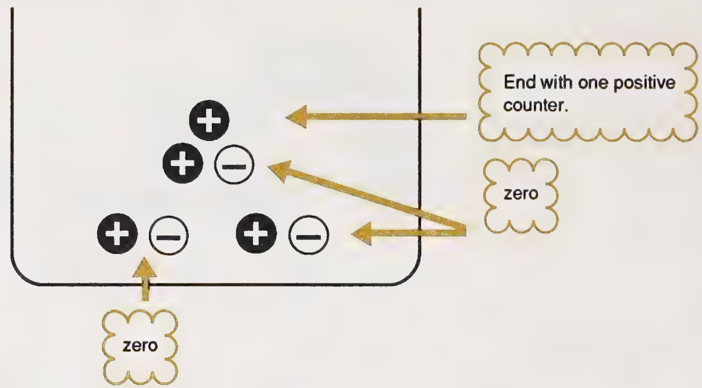
So, $(-2) + (-3) = -5$.

Example 3: What is (+ 4) + (– 3)?

Solution



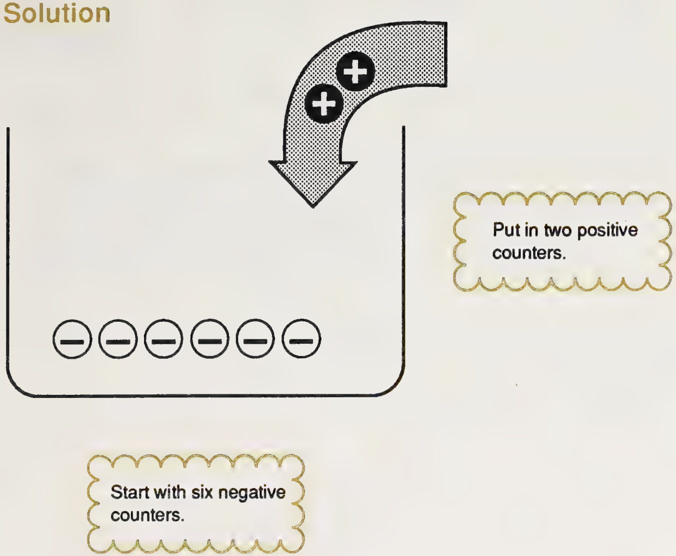
The result is a surplus of one positive counter.



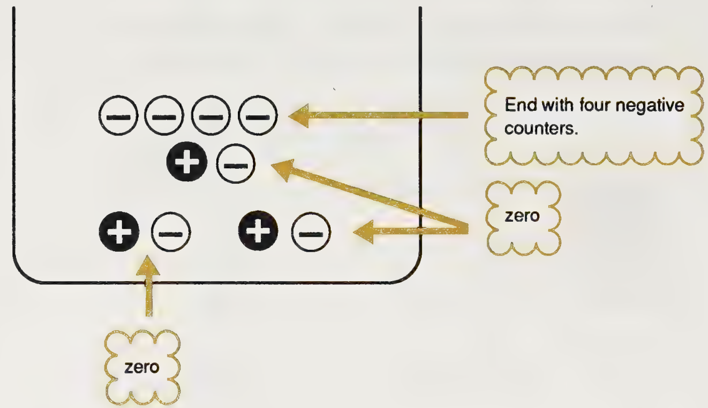
So, (+ 4) + (– 3) = + 1.

Example 4: What is $(-6) + (+2)$?

Solution



The result is a surplus of four negative counters.

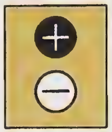


So, $(-6) + (+2) = -4$.

Introductory Activities

Space for Your Work

1. Use your counters to model these sums and complete each addition number sentence.



   a. $(+2) + (+5) =$

   b. $(-3) + (-6) =$

   c. $(-3) + (+6) =$

   d. $(+3) + (-4) =$

   e. $(-2) + (+1) =$

2. What pattern occurs when you add counters with like signs?

3. What pattern occurs when you add counters with unlike signs?

See your learning facilitator to check your answers and to receive further instructions.

Working Together

Adding Integers Without Counters

Now that you understand what happens when you add counters and have discovered a pattern, you should be able to add integers mentally. Use your counters to reinforce the following examples if you wish.

Example 1: Adding integers with like signs

- $(+7) + (+2) = +9$ ← $7 + 2 = 9$

  Both addends are positive, so the answer will be positive.

- $(-5) + (-3) = -8$ ← $5 + 3 = 8$

  Both addends are negative, so the answer will be negative.

Example 2: Adding integers with unlike signs

- $(-6) + (+4) = -2$ ← $6 - 4 = 2$

  There are more negatives than positives, so the answer will be negative.

- $(-1) + (+9) = +8$ ← $9 - 1 = 8$

  There are more positives than negatives, so the answer will be positive.

Example 3: Adding zero and an integer

- $0 + (-5) = -5$
- $0 + (+2) = +2$
- $(-6) + 0 = -6$
- $(+3) + 0 = +3$

Adding zero to the integer does not change the integer.

Practice Activities

Space for Your Work

Print Alternative

1. Find each of these sums mentally. Write your answer.

   a. $(+7) + (+1) = \square$

   b. $(-3) + (-6) = \square$

   c. $(-6) + 0 = \square$

   d. $(-4) + (+9) = \square$

   e. $(+2) + (-2) = \square$

   f. $(-6) + (+3) = \square$

   g. $(+11) + (-8) + (+10) = \square$

   h. $(-1) + (+6) + (-4) + (-2) = \square$

2. Solve the following word problem. Show the number sentence which you used.

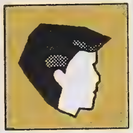
   The temperature at 3 o'clock was 12°C. By 9 o'clock it had fallen 5°C. What was the temperature at 9 o'clock?

Computer Alternative

3. Do Lesson 13 on the disk *Pre-Algebra* from *Computer Drill and Instruction: Mathematics, Level D* (SRA).

   Read the instructions included with the disk before using the program. If you need help, remember to hold down the SHIFT key and press the $\boxed{?}$ key.

See your learning facilitator to check your answers and to receive further instructions.



Working Together

The following examples present everyday events in which integers might be added. **Number lines** are included to help you picture the events and follow the calculations. Use your counters to reinforce the calculations if you wish.

Example

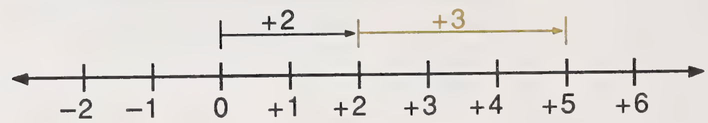
Susan gained 2 kg in weight. Later she gained another 3 kg. How much weight did she gain altogether?

Solution

These events can be described by the following number sentence.

$$(+2) + (+3) = \blacksquare$$

The events can also be represented on a number line.



The black arrow represents the first gain. The coloured arrow represents the second gain. The result is a total gain of 5 kg.

The mental calculation is done as follows.

$$(+2) + (+3) = +5$$

2 + 3 = 5

Both addends are positive, so the answer will be positive.

Example 2

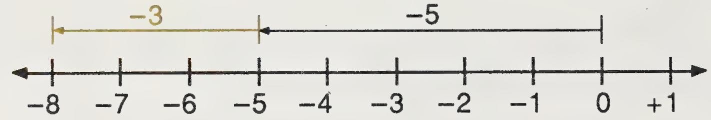
A temperature drop of 5°C was followed by another drop of 3°C. What was the total change in temperature?

Solution

The following number sentence describes the events.

$$(-5) + (-3) = \blacksquare$$

The following number line pictures the events.



The black arrow represents the first temperature drop and the coloured arrow represents the second drop. The result is a total drop of 8°C.

Here is the mental calculation.

$$(-5) + (-3) = -8$$

5 + 3 = 8

Both addends are negative, so the sum will be negative.

Example 3

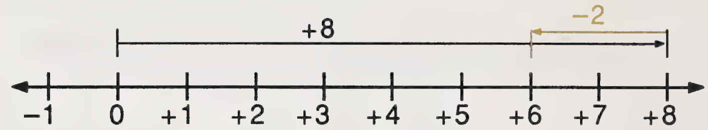
A football team gained eight yards on one play and then lost two yards on the next play. How many yards were gained or lost on the two plays together?

Solution

These events are described by the following number sentence.

$$(+8) + (-2) = ▩$$

The following number line pictures the events.



The black arrow represents the gain on the first play, while the coloured arrow represents the loss on the second play. The result is a net gain of six yards.

The mental calculation is done like this.

$$(+8) + (-2) = +6$$

8 − 2 = 6

There are more positives, so the sum is positive.

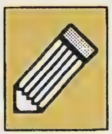
Extra Practice

Space for Your Work

Computer Alternative

1. Work with the program *Integers* from the disk *Integers/Integer Fast Facts* (EduSoft). Read the User's Guide supplied with the disk. Choose the addition operation.

Print Alternative



2. Find each of these sums mentally. Write the answer.

   a. $(-4) + 0 = \blacksquare$

   b. $(+6) + (+3) = \blacksquare$

   c. $(-5) + (+8) = \blacksquare$

   d. $(-1) + (-10) = \blacksquare$

   e. $(+12) + (-7) = \blacksquare$

Space for Your Work

3. Draw a number line to show each of these events.

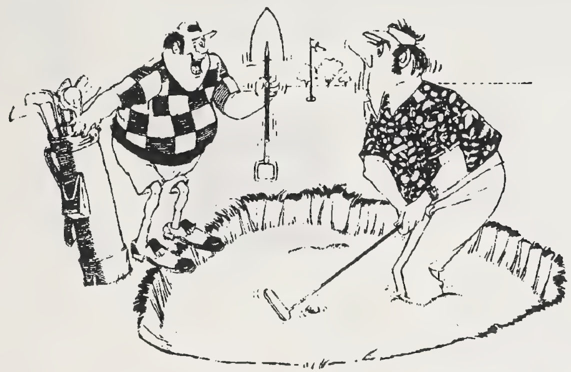
   a. a growth of 2 cm followed by a growth of 5 cm

   b. a temperature rise of 8°C followed by a drop of 5°C

   c. a withdrawal of $5 followed by a deposit of $10

4. Write a number sentence to describe each of the events in Question 3. Give the correct answer.

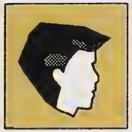
5. Colonel Bogey scores -1 on his first round of golf and $+4$ on the next round. What is his score after both rounds?



6. Felina lost six marbles in one game and then lost 10 more in another game. What was her total loss?

Space for Your Work

See your learning facilitator to check your answers and to receive further instructions.



Working Together

Many calculators have a key which looks like this [+/–].
It is called the sign change key, and it is used to change the sign of a number.

Example

Key Press	Display
[5]	5
[+/–]	–5
[+/–]	5
[+/–]	–5

The sign change key can be used to add integers.

Example 1: What is $(+3) + (-8)$?

Solution

Key Press	Display
[3]	3
[+] [8] [+/–]	–8
[=]	–5

So, $(+3) + (-8) = -5$.

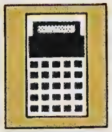
Example 2: What is $(-1) + (+5)$?

Solution

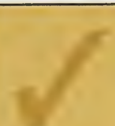
Key Press	Display
[1] [+/–]	–1
[+] [5]	5
[=]	4

So, $(-1) + (+5) = 4$.

Concluding Activities



Use your calculator to complete the *How's Business?* puzzle[1] on the following page.



See your learning facilitator to check your answers and to receive further instructions.

[1]Creative publications for excerpt from *Algebra with Pizzazz*

How's Business?

Each person below is answering the question "How's Business?"

To decode their answers do any question at the right and find your answer in code below. Each time the answer appears in the code, write the letter of that exercise above it. Keep working until you have decoded all four responses.

I (+ 10) + (− 32)	W (+ 54) + (− 73)	I (− 50) + (+ 50)
B (− 15) + (+ 41)	J (+ 83) + (− 53)	Y (+ 737) + (− 923)
V (− 39) + (− 44)	U (− 48) + (+ 85)	N (− 285) + (+ 198)
E (− 27) + (+ 86)	F (− 85) + (+ 48)	C (− 457) + (− 389)
M (+ 61) + (− 12)	P (− 16) + (− 77)	R (+ 95) + (− 93)
K (− 75) + (+ 28)	L (+ 63) + (+ 98)	S (− 95) + (+ 93)
A (− 37) + (− 41)	T (− 105) + (+ 113)	H (+ 69) + (− 12)
D (− 165) + (− 92)		

Soldier Mine is ______.

30	37	− 2	8		− 37	− 22	− 87	59		8	57	− 78	− 87	− 47	− 2

Boxer Mine is ______.

− 78	26	0	37	8		8	0		− 22	49	− 93	2	0	− 83	59

Steak Sauce Maker Mine is ______.

− 19	0	2	− 2	59		8	57	− 22	− 2		− 186	59	− 78	2

Math Teacher Mine is ______.

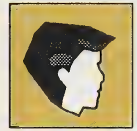
− 846	161	− 78	− 2	− 2	− 186

SUBTRACTING INTEGERS

What Lies Ahead

In this section you will learn these skills.

- subtracting integers using objects
- subtracting integers without using objects



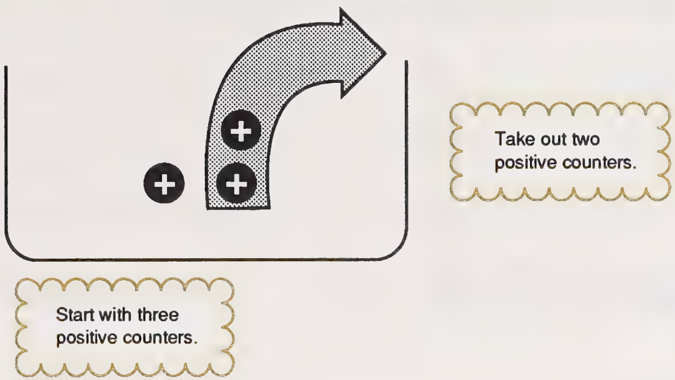
This section continues with integer operations, specifically subtraction. Be alert for an addition to your vocabulary and a new wrinkle in calculations.

Subtracting Integers with Counters

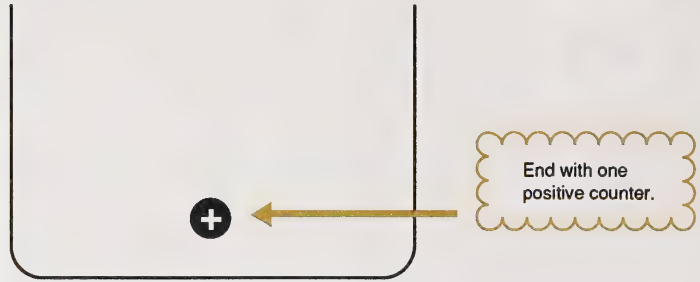
You can think of subtracting integers as taking counters out of a container.

Example 1: What is $(+3) - (+2)$?

Solution



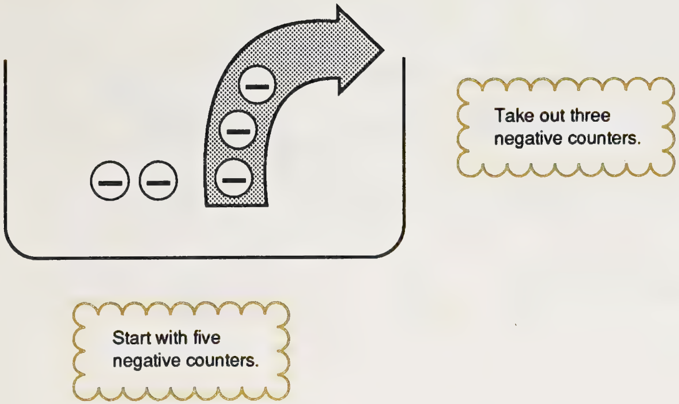
The result is one positive counter left in the container.



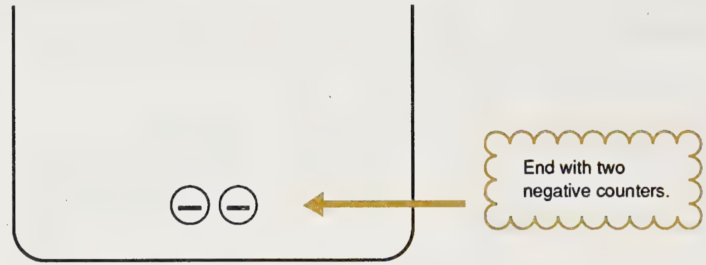
So, $(+3) - (+2) = +1$.

Example 2: What is $(-5) - (-3)$?

Solution



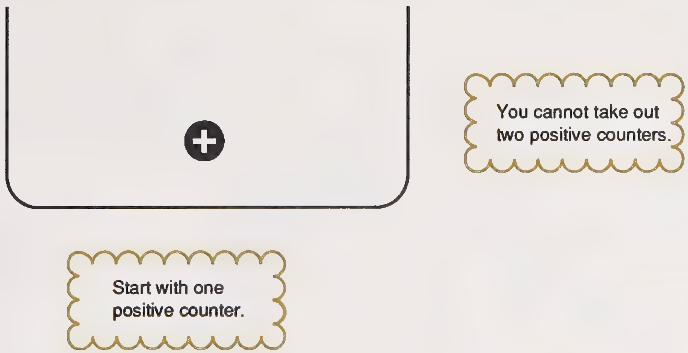
The result is two negative counters left in the container.



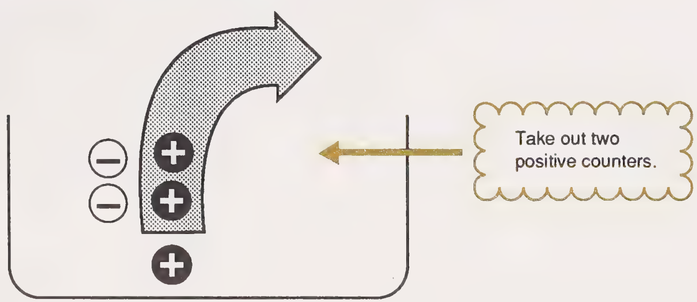
So, $(-5) - (-3) = -2$.

Example 3: What is $(+1) - (+2)$?

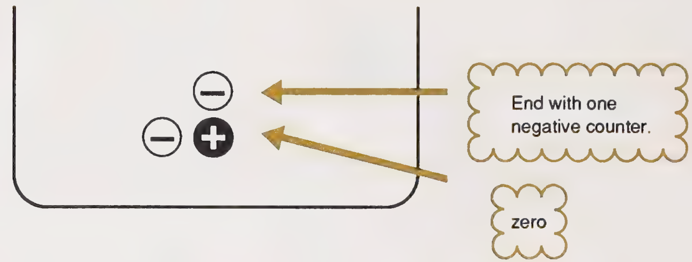
Solution



You cannot take out two positive counters, so put in two pairs of positive and negative counters. This will not change the original charge in the container. Then take out two positive counters.



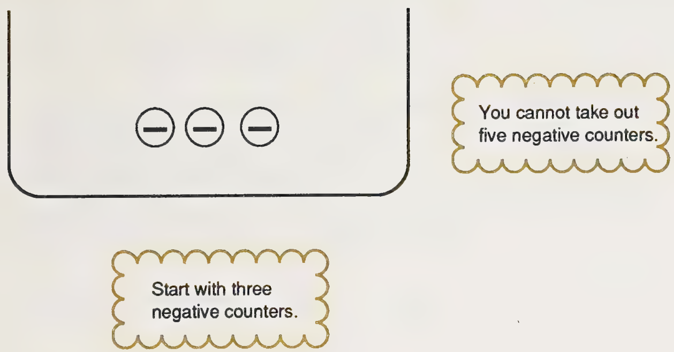
The result is a surplus of one negative counter left in the container.



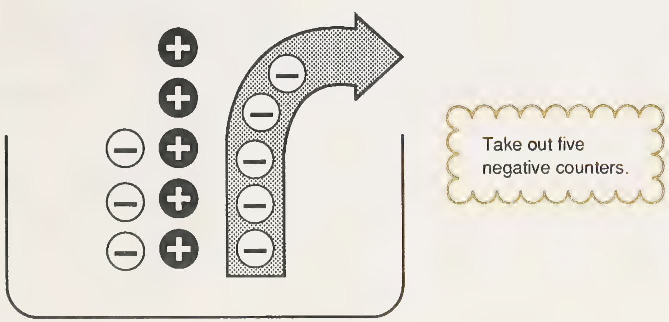
So, $(+1) - (+2) = -1$.

Example 4: What is $(-3) - (-5)$?

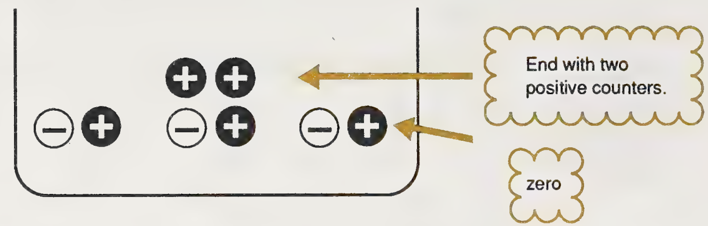
Solution



You cannot take out five negative counters, so put in five pairs of positive and negative counters. This will not change the original charge in the container. Then take out five negative counters.



The result is a surplus of two positive counters left in the container.

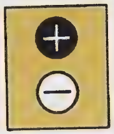


So, $(-3) - (-5) = +2$.

Space for Your Work

Introductory Activities

1. Use your counters to find the differences for each of the following cases.



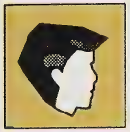
   a. (+ 6) − (+ 3) = ▩
   b. (− 6) − (− 5) = ▩
   c. (+ 3) − (− 3) = ▩
   d. (− 5) − (− 6) = ▩
   e. (− 7) − (+ 4) = ▩
   f. (+ 2) − (− 5) = ▩

2. Use your counters to model the following sums and differences.

   a. (+ 3) − (+ 1) = ▩ and (+ 3) + (− 1) = ▩
   b. (− 2) − (− 1) = ▩ and (− 2) + (+ 1) = ▩
   c. (+ 3) − (− 2) = ▩ and (+ 3) + (+ 2) = ▩

3. What did you notice about the sums and differences in Question 2?

See your learning facilitator to check your answers and to receive further instructions.



Working Together

Subtracting Integers Mentally

From your investigations with counters you learned that subtracting an integer produces the same result as adding that integer's additive inverse. This fact will help you subtract integers mentally. Reinforce the following examples with your counters if you wish.

Example 1: What is $(+4) - (-3)$?

Solution

$$(+4) - (-3) = (+4) + \overset{\text{the additive inverse of } -3}{(+3)}$$

$$= +7$$

Example 2: What is $(+5) - (+6)$?

Solution

$$(+5) - (+6) = (+5) + \overset{\text{the additive inverse of } +6}{(-6)}$$

$$= -1$$

Example 3: What is $(-5) - (+3)$?

Solution

$$(-5) - (+3) = (-5) + \overset{\text{the additive inverse of } +3}{(-3)}$$

$$= -8$$

Example 4: What is $(-3) - (-8)$?

Solution

$$(-3) - (-8) = (-3) + \overset{\text{the additive inverse of } -8}{(+8)}$$

$$= +5$$

Practice Activities

Space for Your Work

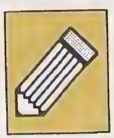
Computer Alternative

1. Do Lesson 14 on the disk *Pre-Algebra* from the package *Computer Drill and Instruction: Mathematics, Level D* (SRA).

   Read the instructions included with the disk before using the program. If you need help, remember to hold down the SHIFT key and press the

   $\boxed{?}$ key.

Print Alternative



2. Fill in the missing integer in each of these cases.

   a. $(+4) - (+4) = (+4) + \square$

   b. $(+7) - (-12) = (+7) + \square$

   c. $(-6) - (-8) = (-6) + \square$

   d. $(-3) - (-9) = (-3) + \square$

   e. $(-5) - (+6) = (-5) + \square$

3. Find the difference in each of the following questions.

   a. $(-40) - (+40) =$ ▒

   b. $(-17) - (-12) =$ ▒

   c. $(+20) - (+8) =$ ▒

   d. $(+13) - 0 =$ ▒

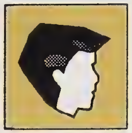
   e. $(+4) - (+6) =$ ▒

4. Solve the following word problem. Use a number sentence to find the answer.

   A golfer's score went from -1 at the end of the first round to $+11$ at the end of the tournament. By how many strokes did his score change?

See your learning facilitator to check your answers and to receive further instructions.

Space for Your Work



Working Together

Everyday events which involve the subtraction of integers are presented in the following examples. Number lines are used to help you picture the events and calculations.

Example 1

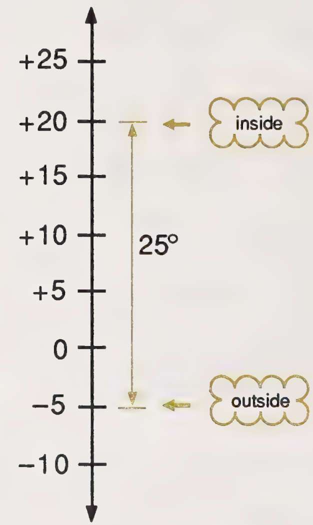
On a winter day the temperature inside a house was + 20°C, while the outside temperature was – 5°C. What was the temperature difference?

Solution

The following number sentence describes the situation.

$$\underset{\text{outside}}{\underset{\uparrow}{(+20)}} - \underset{\text{inside}}{\underset{\uparrow}{(-5)}} = ▩$$

Or, the situation can be pictured on a number line.



The double arrow shows the difference between the two temperatures. The difference is 25°C.

The mental calculation is done as follows.

$$(+20) - (-5) = (+20) + (+5)$$
$$= +25$$

Subtracting an integer produces the same result as adding the integer's additive inverse.

The difference in temperature between the inside of the house and the outside was 25°C.

Example 2

The chart below shows one day's opening and closing share prices.

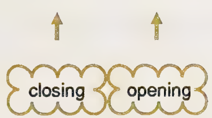
	Opening	Closing
Allison Products	\$5	\$2

Show the difference in closing and opening prices, and tell whether the changes are gains or losses.

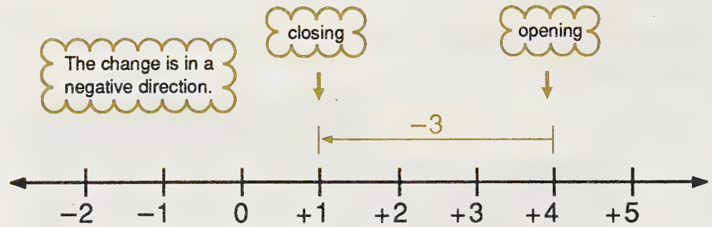
Solution

The following number sentence describes the situation.

$$(+2) - (+5) = \blacksquare$$



The situation can also be shown on a number line.



The mental calculation is done as follows.

$$\begin{aligned}(+2) - (+5) &= (+2) + (-5) \\ &= -3\end{aligned}$$

The result for Allison Products is a negative change or a loss of \$3.

Example 3

The chart below shows one day's opening and closing share prices.

	Opening	Closing
Beverly Ltd.	$7	$9

Show the difference in closing and opening prices, and tell whether the changes are gains or losses.

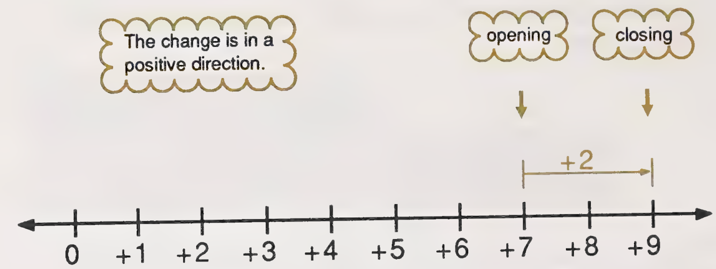
Solution

The following number sentence describes the situation.

$(+9) - (+7) = \blacksquare$

closing opening

The situation can also be shown on a number line.



The mental calculation is done as follows.

$$(+9) - (+7) = 9 + (-7)$$
$$= +2$$

The result for Beverly Ltd. is a positive change or a gain of $2.

Extra Practice

Space for Your Work

1. For each of these integers give the additive inverse.

   a. -7

   b. $+2$

   c. $+10$

   d. -178

   e. $-7\,000$

2. Rewrite each of the following using the **opposite operation** and the additive inverse. Then find the answer.

   a. $(-6) - (+3) = \blacksquare$

   b. $(+5) - (+6) = \blacksquare$

   c. $(-2) - (-4) = \blacksquare$

   d. $(+12) - (-6) = \blacksquare$

Computer Alternative

Space for Your Work

3. Use the program *Integers* from the disk *Integers/Integer Fast Facts* (EduSoft).

   Read the User's Guide supplied with the disk. Choose the subtraction option.

Print Alternative

Use number lines to help you solve Questions 4 to 6.

4. A chinook in southern Alberta raised the temperature from – 18°C to – 3°C. By how much did the temperature change?

5. The peak of Mt. Everest is + 8 848 m or 8 848 m above sea level. The floor of Death Valley is 8 934 m lower than the peak of Mt. Everest. What is the elevation of the floor of Death Valley?

6. The Pacific Ocean floor is – 11 034 m or 11 034 m below sea level. The Dead Sea floor is – 400 m or 400 m below sea level. What is the difference between their elevations?

See your learning facilitator to check your answers and to receive further instructions.

Space for Your Work

Working Together

You learned about the sign change key $\boxed{+/-}$ which changes the sign of a number.

Example

Key Press	Display
$\boxed{3}$	3
$\boxed{+/-}$	–3
$\boxed{+/-}$	3
$\boxed{+/-}$	–3

The sign change key can be used to subtract integers.

Example 1: (– 2) – (+ 7)

Solution

Key Press	Display
$\boxed{2}\boxed{+/-}$	–2
$\boxed{-}\boxed{7}$	7
$\boxed{=}$	–9

So, (– 2) – (+ 7) = – 9.

Example 2: (– 4) – (– 6)

Solution

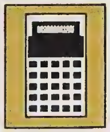
Key Press	Display
$\boxed{4}\boxed{+/-}$	–4
$\boxed{-}\boxed{6}\boxed{+/-}$	–6
$\boxed{=}$	2

So, (– 4) – (– 6) = + 2.

Concluding Activities

Space for Your Work

Use your calculator to solve Questions 1 to 3.



1. The deepest spot in the world is Challenger Deep in the Mariana Trench off the Asian coast. It is 11 033 m below sea level. The highest spot is Mt. Everest in Nepal. It is 8 848 m in height. How far apart are the two spots vertically?

2. The highest ever recorded temperature on Earth is 58°C. The lowest ever is –89°C. What is the difference between the highest and lowest temperatures recorded on Earth?

3. Complete the puzzle on the following page.

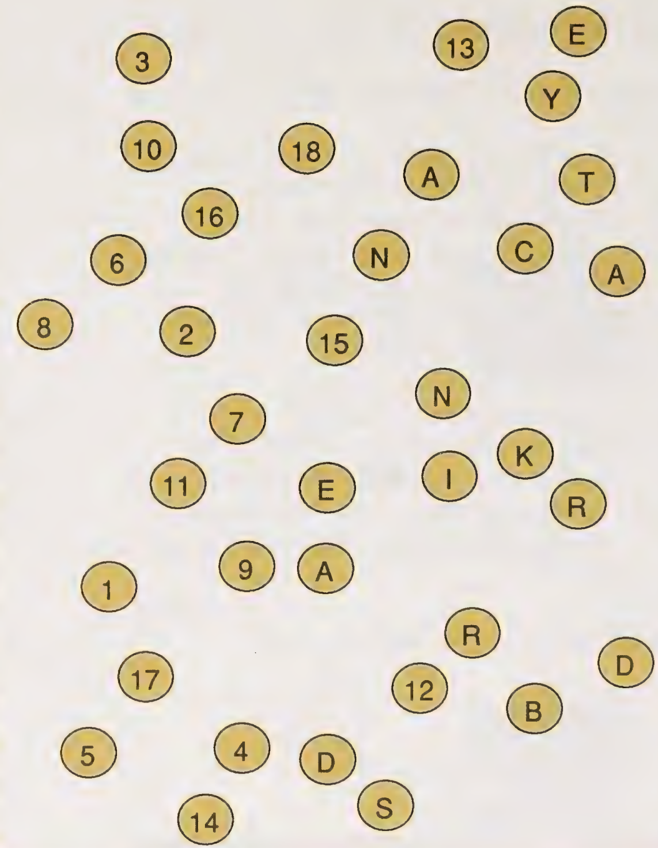
See your learning facilitator to check your answers and to receive further instructions.

What Do Vampires Do on Halloween?

To answer this question, follow these directions.

Draw a straight line connecting each exercise with its correct answer. Each line will cross a number and a letter. The number tells you where to put the letter in the row of boxes at the bottom of the page.

(+ 15) − (− 6)		− 9
(− 15) − (+ 6)		− 113
(− 15) − (− 6)		− 53
(+ 15) − (+ 6)		120
(− 83) − (+ 30)		21
(− 83) − (− 30)		53
(+ 30) − (− 83)		− 24
(− 30) − (− 83)		0
(+ 27) − (+ 54)		− 21
(− 78) − (− 78)		95
(+ 60) − (− 60)		113
(− 47) − (+ 77)		− 95
(− 47) − (− 77)		− 124
(+ 19) − (+ 43)		9
(− 36) − (− 18)		− 27
(+ 55) − (− 40)		30
(− 40) − (+ 55)		− 15
(+ 40) − (+ 55)		− 18



1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18

MULTIPLYING INTEGERS

What Lies Ahead

In this section you will learn to multiply integers with and without objects.

Working Together

You have already learned to add and subtract integers. Multiplication is really repeated addition. Keep this in mind as you learn to multiply integers.

If the sign of the first factor is positive, think of this as adding groups of counters to the container.

- $(+2)(-3)$ means to put two groups of three negative counters into the container.
- $(+2)(+5)$ means to put two groups of five positive counters into the container.

If the sign of the first factor is negative, think of this as subtracting groups of counters from the container.

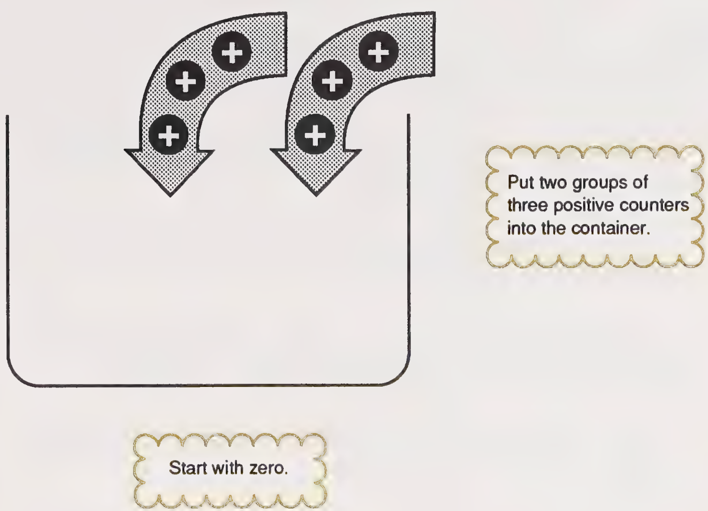
- $(-3)(-2)$ means to take three groups of two negative counters out of the container.
- $(-3)(+5)$ means to take three groups of five positive counters out of the container.

Multiplying Integers with Counters

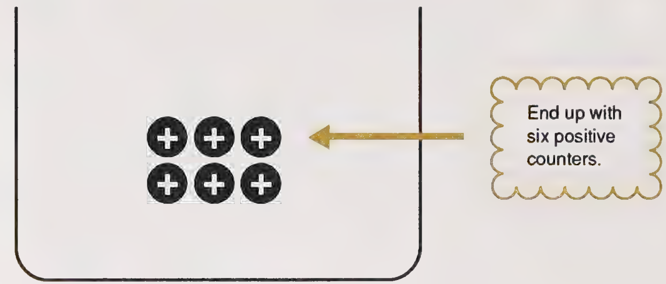
Example 1: What is $(+2) \times (+3)$?

Solution

$(+2) \times (+3)$ means to put two groups of three positive counters into the container.



The result is six positive counters in the container.

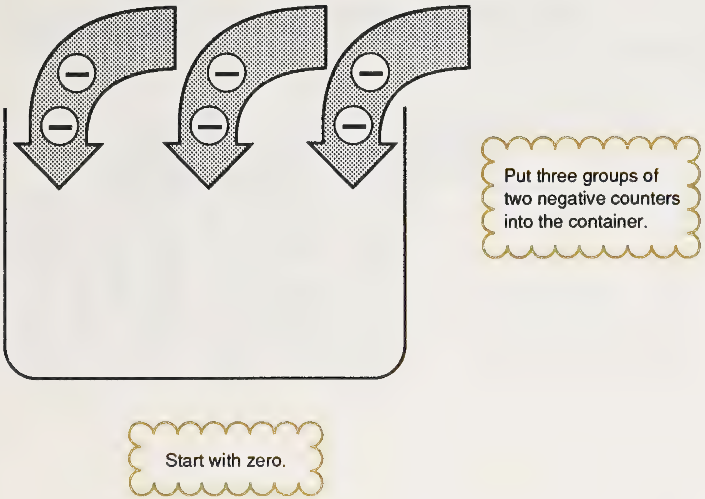


So, $(+2) \times (+3) = +6$.

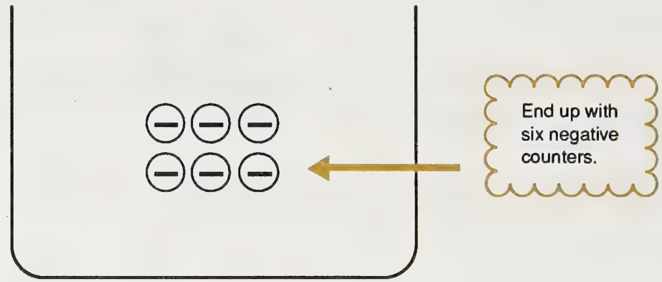
Example 2: What is $(+3) \times (-2)$?

Solution

$(+3) \times (-2)$ means to put three groups of two negative counters into the container.



The result is six negative counters in the container.



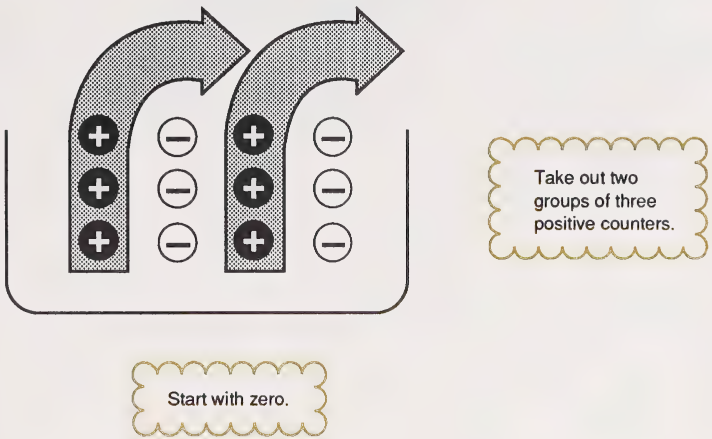
So, $(+3) \times (-2) = -6$.

Example 3: What is $(-2) \times (+3)$?

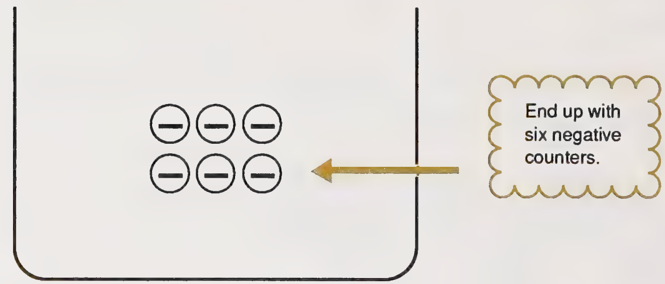
Solution

$(-2) \times (+3)$ means to take two groups of three positive counters out of the container.

You must first put in pairs of positive and negative counters. This will not change the charge in the containers.



The result is six negative counters in the container.



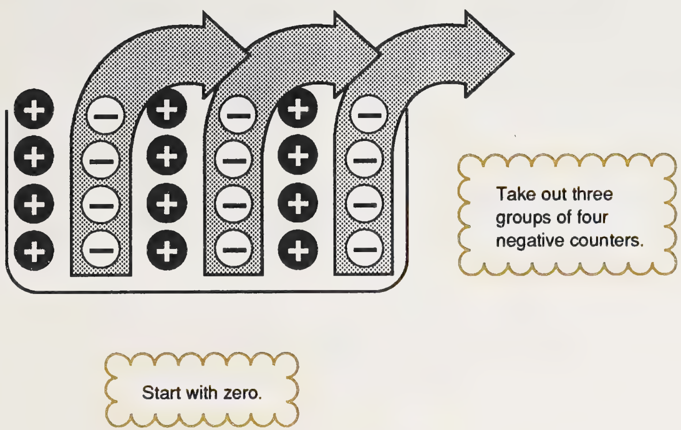
So, $(-2) \times (+3) = -6$.

Example 4: What is $(-3) \times (-4)$?

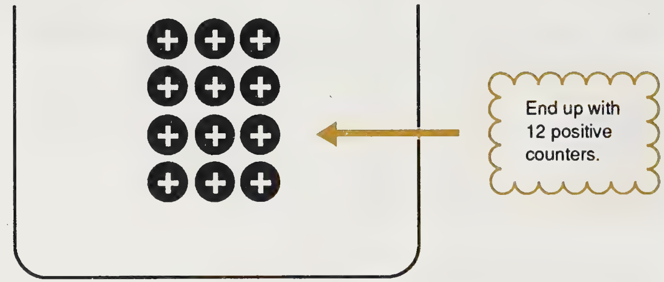
Solution

$(-3) \times (-4)$ means to take three groups of four negative counters out of the container.

You must first put in pairs of positive and negative counters. This will not change the charge in the container.



The result is 12 positive counters in the container.

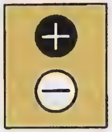


So, $(-3) \times (-4) = +12$.

Introductory Activities

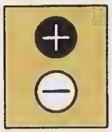
Space for Your Work

1. Use counters to model the following products. Write a number sentence which shows the answer.



   a. $(+2) \times (+4)$
   b. $(+3) \times (-3)$
   c. $(-2) \times (-4)$
   d. $(-3) \times (+1)$

2. Use counters to model the following and find a pattern. Watch the signs.



   a. $(+3) \times (+1)$
   b. $(+2) \times (-4)$
   c. $(+1) \times (+2)$
   d. $(-1) \times (-4)$
   e. $(-2) \times (-1)$
   f. $(-3) \times (+2)$

3. What do you notice about the product of two factors with like signs?

4. What do you notice about the product of two factors with unlike signs?

See your learning facilitator to check your answers and to receive further instructions.

Working Together

Multiplying Integers Mentally

By modelling with counters you were able to discover a pattern that will help you to multiply integers mentally. That is, two like signs make a positive product and two unlike signs make a negative product. Reinforce the following examples with counters if you wish.

Example 1: What is $(+5) \times (+6)$?

Solution

$$(+5) \times (+6) = +30 \leftarrow \boxed{5 \times 6 = 30}$$

The signs are like so the product is positive.

Example 2: What is $(-3) \times (-8)$?

Solution

$$(-3) \times (-8) = +24 \leftarrow \boxed{3 \times 8 = 24}$$

The signs are like so the product is positive.

Example 3: What is $(+8) \times (-2)$?

Solution

$$(+8) \times (-2) = -16 \leftarrow \boxed{8 \times 2 = 16}$$

The signs are unlike so the product is negative.

Example 4: What is $(-7) \times (+5)$?

Solution

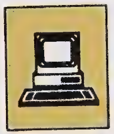
$$(-7) \times (+5) = -35 \leftarrow \boxed{7 \times 5 = 35}$$

The signs are unlike so the product is negative.

Practice Activities

Space for Your Work

Computer Alternative



1. Do Lesson 15 on the disk *Pre-Algebra* from the package *Computer Drill and Instruction: Mathematics, Level D* (SRA).

   Read the instructions included with the disk before using the program. If you need help, remember to hold down the SHIFT key and press the

   $\boxed{?}$ key.

Print Alternative

Space for Your Work

2. Tell which products will be positive and why.

   a. $(-1) \times (+3)$

   b. $(-4) \times (-4)$

   c. $(+5) \times (-2)$

   d. $(+8) \times 0$

   e. $(+2) \times (-6)$

   f. $(+3) \times (+9)$

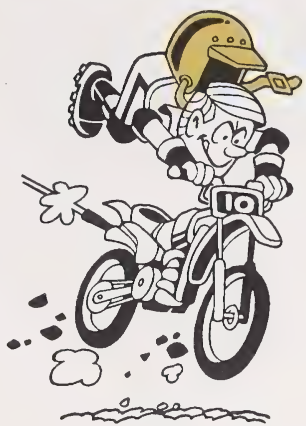
   g. $(-7) \times (+5)$

   h. $(-4) \times (-10)$

3. Find the products for Question 2.

4. In one week five drivers in Hooterville each had three demerit points or + 3 charged to their licences. What was the total number of demerits for the week?

5. In a motorcross meet Kim was charged with seven penalty points or – 7 in each of the two races he ran. What was his total penalty?



Space for Your Work

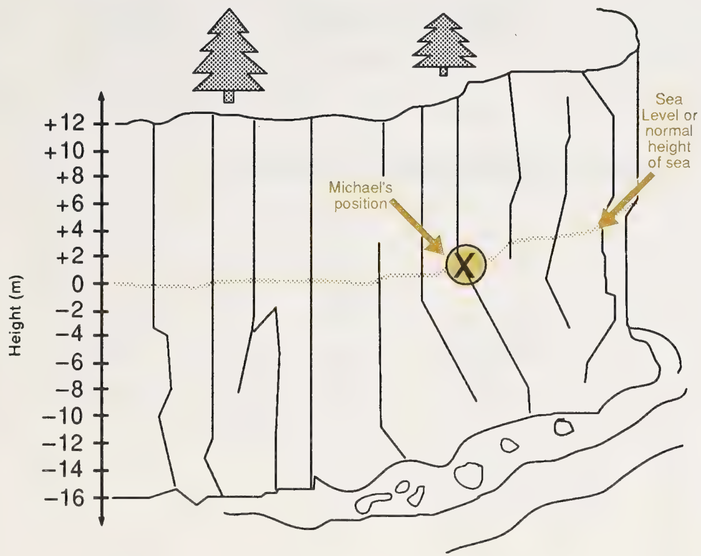
See your learning facilitator to check your answers and to receive further instructions.

Working Together

Everyday events which involve the multiplication of integers are presented in the following example.

Example

Michael likes to rock climb on the cliffs of the Bay of Fundy in Nova Scotia when the tide is out.



Integers can be used to describe the location of objects.

- Sea level or the starting point, 0 m.
- The metres above sea level are positive.
- The metres below sea level are negative.

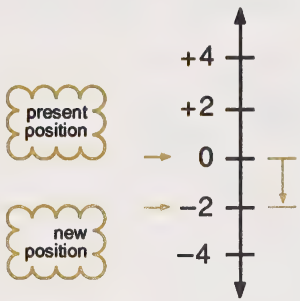
In this diagram note the position of the following.

- Michael is at 0 m or at sea level.
- The top of the cliff is + 12 m.
- The beach is at – 16 m.

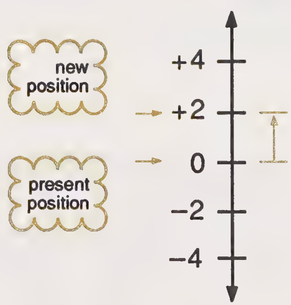
Integers can also be used to indicate change in position.

Michael is at 0 m or at sea level. Climbing up would be in a positive direction. Climbing down would be in a negative direction.

- If Michael climbs down 2 m, his new position will be – 2 m.



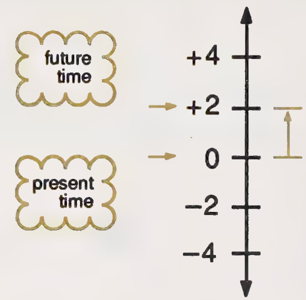
- If Michael climbs up 2 m, his new position will be + 2 m.



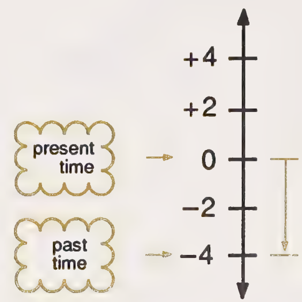
Integers can also be used to indicate change in time.

The time is presently zero minutes. Time in the future is positive. Time in the past is negative.

- Two minutes from now is + 2 minutes.



- Four minutes ago was – 4 minutes.



Example

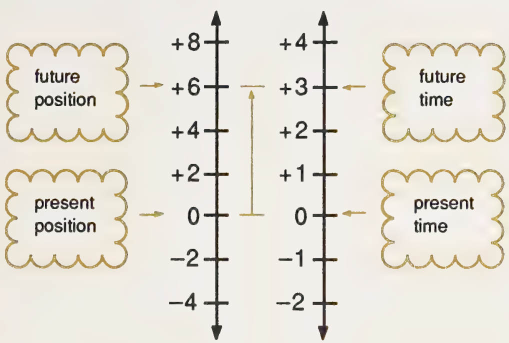
Michael's present position is 0 m and he is climbing up the cliff. If he climbs 2 m in one minute, what will his position be in three minutes?

The following number sentence describes the event.

$$(+3) \times (+2) = \blacksquare$$

number of future minutes | distance up in one minute | future position

The number line below pictures the event.



The mental calculation is as follows.

$$(+3) \times (+2) = \blacksquare$$

$$(+3) \times (+2) = +6$$

3 × 2 = 6

The signs are like, so the product is positive.

In three minutes, Michael will be 6 m above his present position, or 6 m above sea level.

Extra Practice

Space for Your Work

Computer Alternative

1. Use the program *Integers* from the disk *Integers/Integer Fast Facts* (EduSoft).

   Read the User's Guide. Choose the multiplication operation.

Print Alternative

Use number lines to help you solve Questions 2 to 4.

2. Michael's present position is 0 m and he is moving up. If he climbs 2 m in one minute, what was his position five minutes ago?

Here is a number sentence descibing the event.

$$(-5) \times (+2) = \blacksquare$$

number of past minutes | distance up in one minute | past position

Draw a number line for the event and solve the problem using mental calculation.

Space for Your Work

3. Michael's present position is 0 m and he is moving down the cliff. If he moves 2 m in one minute, what will his position be in four minutes?

This number sentence describes the event.

$$(+4) \times (-2) = ▩$$

number of future minutes

distance down in one minute

future position

Draw a number line for the event and solve the problem using mental calculation.

Space for Your Work

4. Michael's present position is 0 m and he is moving down the cliff. If he moves 2 m in one minute, what was his position three minutes ago?

This number sentence describes the events.

$$\underset{\substack{\uparrow\\\text{number of}\\\text{past minutes}}}{(-3)} \times \underset{\substack{\uparrow\\\text{distance down}\\\text{in one minute}}}{(-2)} = \underset{\substack{\uparrow\\\text{past}\\\text{position}}}{\blacksquare}$$

Draw a number line to describe the event, and solve the problem using mental calculation.

See your learning facilitator to check your answers and to receive further instructions.

Space for Your Work

Working Together

You can use Droopy and number lines to help you find any product of integers.

Droopy tackles multiplication of integers by using the following rules.

- He always starts on the zero coordinate on the number line.
- He has two sweaters, one with a positive sign (+), the other with a negative sign (–). When Droopy wears his positive sweater, he faces in the positive direction on the number line. When Droopy wears his negative sweater, he faces in the negative direction on the number line.

- The sign of the first factor determines which sweater he will wear and thus the way he will face. If the sign of the first factor is positive he wears his positive sweater and faces to your right.

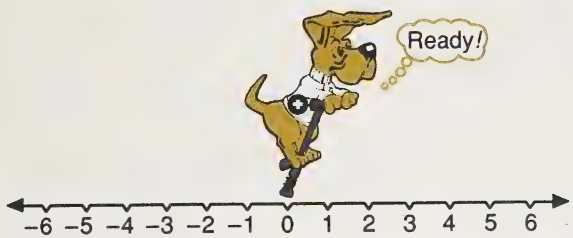
If the sign of the first factor is negative, Droopy puts on his negative sweater and faces in the negative direction on the number line, that is to your left.

- The sign of the second factor determines which way Droopy moves. If the sign of the second factor is positive, he moves forward. If the sign of the second factor is negative, he moves backward.

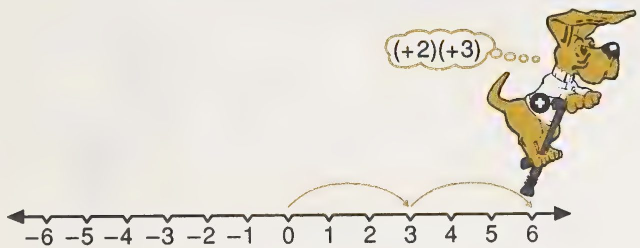
[1] National Council of Teachers of Mathematics for excerpts from Arithmetic Teacher, December 1976, Reston, Virginia.

Example 1: What is $(+2) \times (+3)$?

Since the sign of the first factor is positive, place the positive Droopy on the number line at zero facing right.

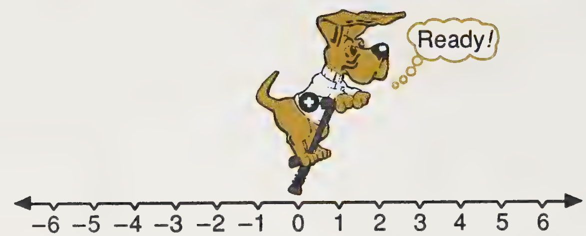


The sign of the second factor is positive, therefore move Droopy forward, which in this case is in the positive direction, two steps of 3. This gives the product + 6.

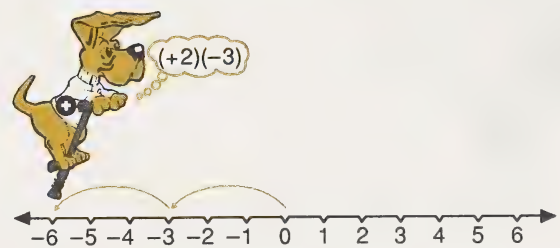


Example 2: What is $(+2) \times (-3)$?

Since the sign of the first factor is positive, place the positive Droopy on the number line at zero facing right.

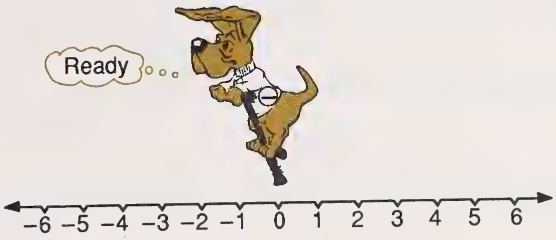


As the sign of the second factor is negative, move Droopy backward, which would be in the negative direction, two steps of 3. Droopy lands at the point − 6, which is the product of $(+2)(-3)$.

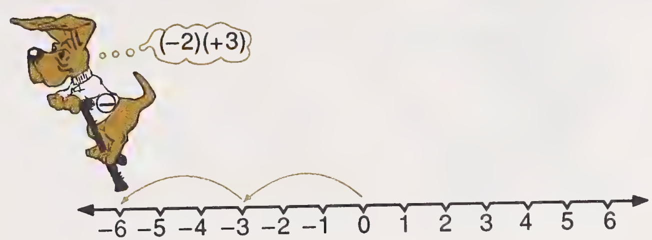


Example 3: What is $(-2) \times (+3)$?

Since the sign of the first factor is negative, place the negative Droopy on the number line at zero facing left.

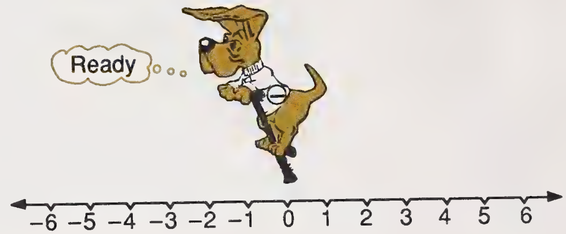


Since the sign of the second factor is positive, move Droopy two steps of 3 in a forward direction, which would be to the left, since he is facing left now. Droopy lands on -6 which is the product of $(-2)(+3)$.

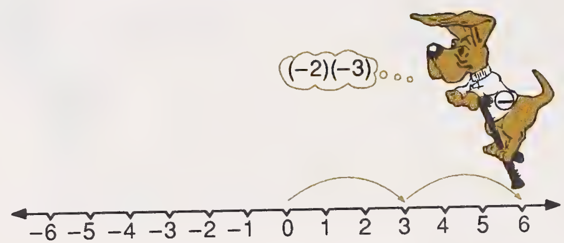


Example 4: What is $(-2) \times (-3)$?

Since the sign of the first factor is negative, place the negative Droopy on the number line at zero facing left.



Since the sign of the second factor is negative, move Droopy two steps of 3 in a backward direction, which would be to the right, since he is facing left now. Droopy lands on $+6$ which is the product of $(-2) \times (-3)$.



Further Practice

Space for Your Work

Find the product. Use the cutout of Droopy from the appendix and number lines to help you.

1. $(-9) \times (+3) =$ ▩
2. $(+1) \times (+12) =$ ▩
3. $(+6) \times (-6) =$ ▩
4. $0 \times (+8) =$ ▩
5. $(-7) \times (-3) =$ ▩
6. $(+5) \times (-9) =$ ▩

See your learning facilitator to check your answers and to receive further instructions.

Working Together

You can multiply both positive and negative integers with your calculator by using the [+/-] key.

Example 1: What is $(-4) \times (+3)$?

Solution

Key Press	Display
4 +/-	–4
× 3	3
=	–12

So, $(-4) \times (+3) = -12$.

Example 2: What is $(-6) \times (-4)$?

Solution

Key Press	Display
6 +/-	–6
× 4 +/-	–4
=	24

So, $(-6) \times (-4) = 24$.

Example 3: What is $(-2) \times (-3) \times (-5)$?

Solution

Key Press	Display
2 +/-	–2
× 3 +/-	–3
× 5 +/-	–5
=	–30

So, $(-2) \times (-3) \times (-5) = -30$.

Example 4: What is $(-1) \times (+2) \times (-3)$?

Solution

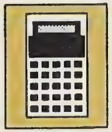
Key Press	Display
1 +/-	–1
× 2	2
× 3 +/-	–3
=	6

So, $(-1) \times (+2) \times (-3) = 6$.

Concluding Activities

Space for Your Work

1. Find the products for each of the following using a calculator.



   a. $(+3) \times (+4) =$ ▒

   b. $(-4) \times (+2) =$ ▒

   c. $(-5) \times (-3) =$ ▒

   d. $(+4) \times (+3) \times (-2) =$ ▒

   e. $(+4) \times (-3) \times (-2) =$ ▒

   f. $(-4) \times (-3) \times (-2) =$ ▒

   g. $(-4) \times (-3) \times (-2) \times (-1) =$ ▒

2. What patterns did you notice in Question 1?

   a. When there are all positive signs, what sign goes with the answer?

   b. When there is an even number of negative signs, what sign goes with the answer?

   c. When there is an odd number of negative signs, what sign goes with the answer?

Space for Your Work

3. Write **P** if the answer will be positive and **N** if the answer will be negative.

   a. $(-4) \times (+2) \times (+9)$

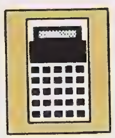
   b. $(+4) \times (+2) \times (+3)$

   c. $(-1) \times (-6) \times (+4)$

   d. $(-5) \times (-2) \times (-6)$

   e. $(+2) \times (-3) \times (+5) \times (-2)$

Use a calculator for Questions 4 and 5.



4. Find the answers to Question 3.

5. Complete the puzzle on the next page[1].

See your learning facilitator to check your answers and to receive further instructions.

[1] Creative Publications for excerpts from *Algebra with Pizzaz*.

Hidden Message

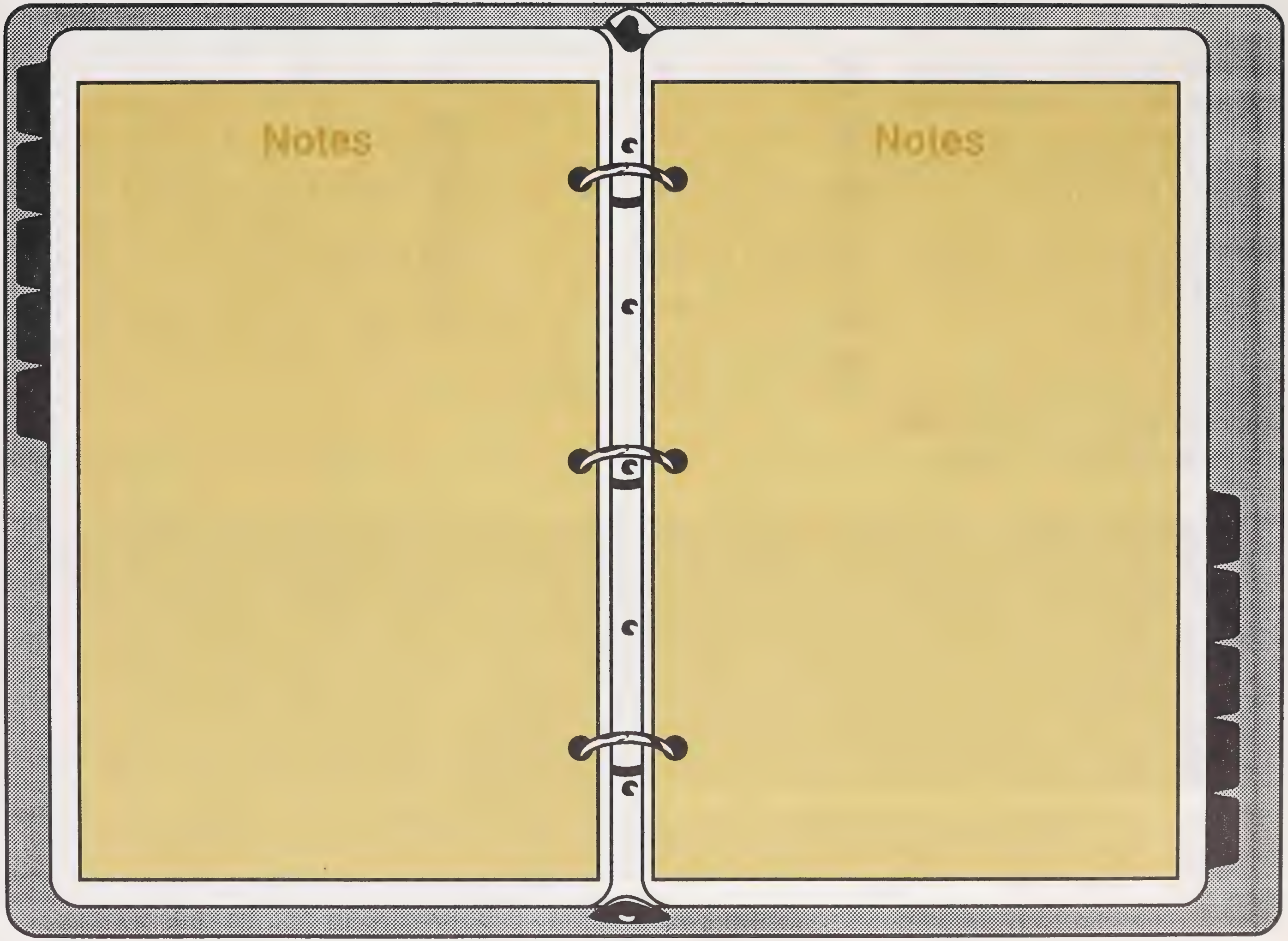
First, do each exercise at the right and find your answer in the rectangle below. The correct answers run across from left to right. These answers may take up any number of boxes in a row.

Second, shade in the boxes containing each correct answer. When you finish, there will be 27 boxes not shaded in.

Starting on the top line and working from left to right, print the 27 letters that remain in the boxes at the bottom of the page. A hidden message will appear!

1. $(-48) \times (-17)$
2. $(+39) \times (-68)$
3. $(-8) \times (+4) \times (-7)$
4. $(-6) \times (+1589)$
5. $(-15) \times (-25) \times (-35)$
6. $(-100) \times (+9) \times (+53)$
7. $(-3) \times (+8) \times (-9) \times (-7)$
8. $(-94) \times (-1) \times (+78) \times (+20)$
9. $(-8) \times (-8) \times (-8) \times (-8)$
10. $(-498) \times (+10) \times (+20) \times (+30)$
11. $(-3) \times (-3) \times (-3) \times (-3) \times (-3)$

J -1	U 8	M 1	A 6	O 4	M -2	I 9	S 8	P 8	T 0	H 0	O 0	G 3	G 9	A -1	M 5	E 1	A 2	E -1	R 7	O 2	B 2	A 4	S 0	D 7
O -9	B 3	E 6	Y -2	L 6	I 5	A 2	T 4	T -1	E 0	N -1	T 3	I 1	O 2	N 5	R -8	O -2	L 4	E 3	I 4	N 4	G 0	O 9	O 6	N 7
T -3	H 2	A -9	T 5	O 3	T 4	E 9	L 4	O -3	P 1	H 4	Q 6	U 6	B 4	O 0	N 5	G -6	R 0	E -4	A 7	T 7	A 0	S 0	U 4	N 3


Notes
Notes

DIVIDING INTEGERS

What Lies Ahead

In this section you will learn these skills.

- dividing integers using objects
- dividing integers without using objects

Working Together

In the previous section you learned how to multiply integers. Dividing and multiplying are closely related. Keep this in mind as you learn to divide integers in this section.

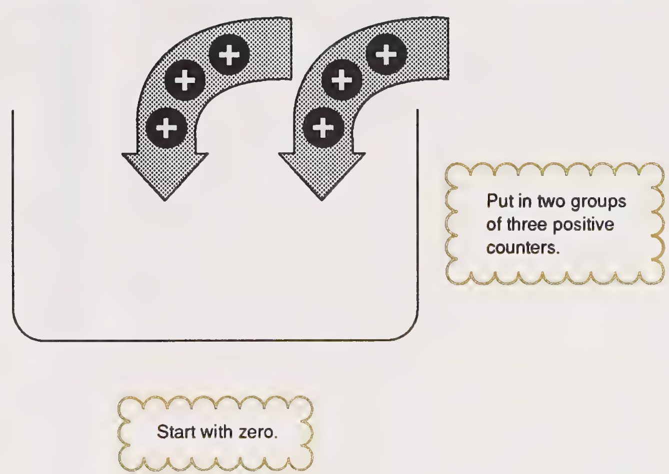
Dividing Integers with Counters

Example 1: What is $(+6) \div (+2)$?

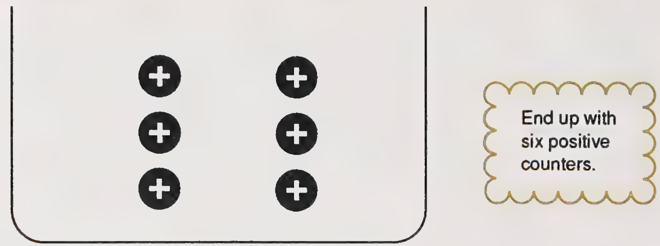
Solution

$(+6) \div (+2) = \blacksquare$ is the same as $(+2) \times \blacksquare = +6$.

So, to get six positive counters in the container, put in two groups of how many?



The result is six positive counters in the container.



How many counters were in each group that you put into the container?

Each group had three positive counters.

So, $(+6) \div (+2) = +3$.

Check

$$(+2) \times (+3) = +6$$

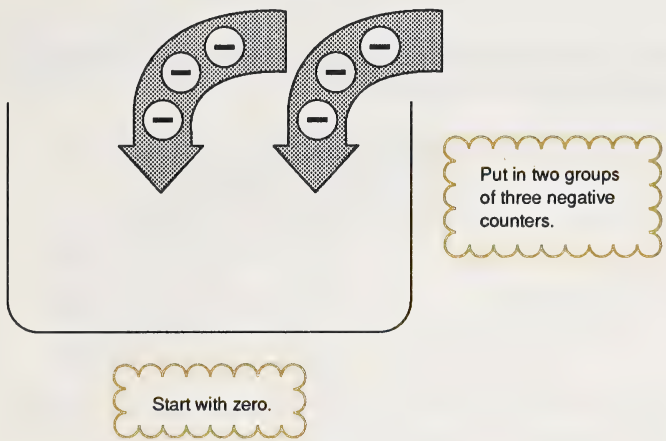
Putting in two groups of three positive counters results in six positive counters in the container.

Example 2: What is $(-6) \div (+2)$?

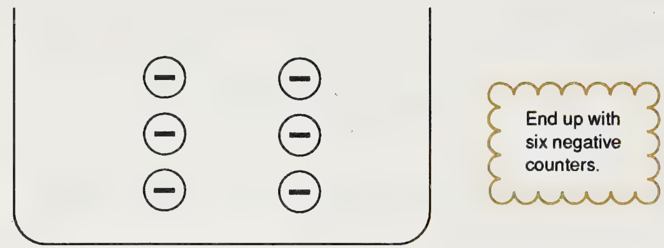
Solution

$(-6) \div (+2) = \blacksquare$ is the same as $(+2) \times \blacksquare = -6$.

So, to get six negative counters in the container, you have to put in two groups of how many?



The result is six negative counters in the container.



How many counters were there in each group that you put into the container?

Each group had three negative counters.

So, $(-6) \div (+2) = -3$.

Check

$$(+2) \times (-3) = -6$$

Putting in two groups of three negative counters results in six negative counters in the container.

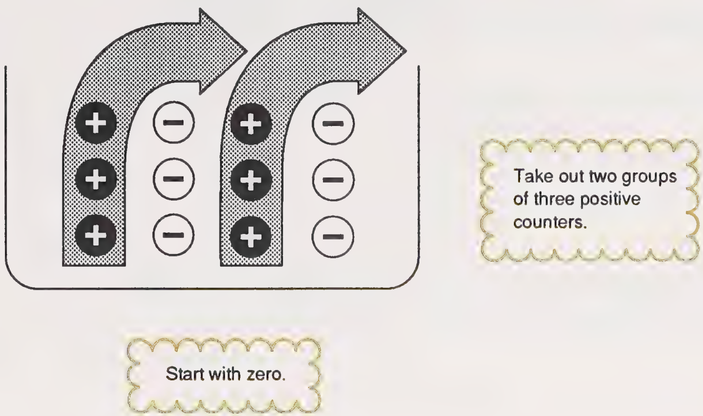
Example 3: What is $(-6) \div (-2)$?

Solution

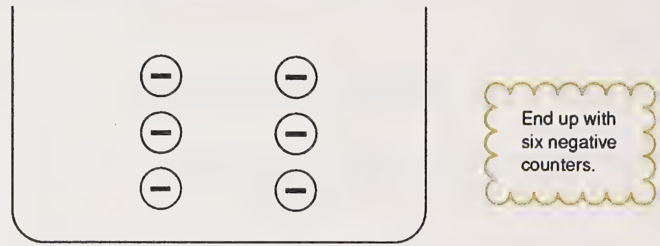
$(-6) \div (-2) = \blacksquare$ means the same as
$(-2) \times \blacksquare = -6$

So, to get six negative counters in the container, take out two groups of how many?

You must first put in pairs of positive and negative counters. This will not change the charge in the container.



The result is six negative counters in the container.



How many counters were there in each group that was taken out of the container?

Each group had three positive counters.

So, $(-6) \div (-2) = +3$

Check

$$(-2) \times (+3) = -6$$

Taking out two groups of three positive counters results in six negative counters in the container.

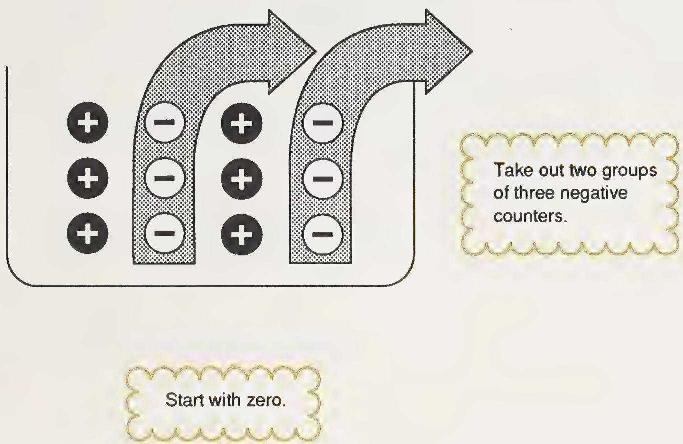
Example 4: What is $(+6) \div (-2)$?

Solution

$(+6) \div (-2) = \blacksquare$ means the same as
$(-2) \times \blacksquare = +6$.

To get six positve counters in the container, take out two groups of how many?

You must first put in pairs of positive and negative counters. This will not change the charge in the container.



The result is six positive counters in the container.

How many counters were there in each group that was taken out of the container?

Each group had three negative counters.

So, $(+6) \div (-2) = -3$.

Check

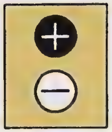
$$(-2) \times (-3) = +6$$

Taking out two groups of three negative counters results in six positive counters in the container.

Introductory Activities

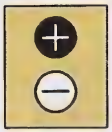
Space for Your Work

1. Use counters to model the following and find the result.



   a. $(+6) \div (+3) =$ ▩
   b. $(+12) \div (-3) =$ ▩
   c. $(-8) \div (+4) =$ ▩
   d. $(-8) \div (-2) =$ ▩

2. Use counters to model the following and find a pattern. Watch the signs.



   a. $(+4) \div (+2) =$ ▩
   b. $(+6) \div (-3) =$ ▩
   c. $(-2) \div (+2) =$ ▩
   d. $(-8) \div (-4) =$ ▩
   e. $(+9) \div (-3) =$ ▩
   f. $(-3) \div (-1) =$ ▩

3. What do you notice about the quotient of two numbers with like signs?

4. What do you notice about the quotient of two numbers with unlike signs?

See your learning facilitator to check your answers and to receive further instructions.

Working Together

Dividing Integers Mentally

From the Introductory Activities you were able to discover a pattern that will help you divide integers mentally. If the numbers have like signs, the quotient is positive. If the numbers have unlike signs, the quotient is negative.

Example 1: What is $(+10) \div (+5)$?

Solution

$(+10) \div (+5) = +2$ ← $10 \div 5 = 2$

The signs are like so the answer is positive.

Example 2: What is $(-12) \div (-6)$?

Solution

$(-12) \div (-6) = +2$ ← $12 \div 6 = 2$

The signs are like so the answer is positive.

Example 3: What is $(+8) \div (-4)$?

Solution

$(+8) \div (-4) = -2$ ← $8 \div 2 = 4$

The signs are unlike so the answer is negative.

Example 4: What is $(-6) \div (+3)$?

Solution

$(-6) \div (+3) = -2$ ← $6 \div 3 = 2$

The signs are unlike so the answer is negative.

Practice Activities

Space for Your Work

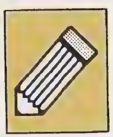
Computer Alternative

1. Do Lesson 16 on the disk *Pre-Algebra* from the package *Computer Drill and Instruction: Mathematics, Level D* (SRA).

   Read the instructions included with the disk before using the program. If you need help, remember to hold down the SHIFT key and press the

   [?] key.

Print Alternative



2. Provide the correct sign to make each number sentence true. Why did you choose the sign that you used?

   a. (+ 56) ÷ (+ 7) = ▒ 8

   b. (– 121) ÷ (– 11) = ▒ 11

   c. (+ 1272) ÷ (– 24) = ▒ 53

   d. (– 7259) ÷ (+ 119) = ▒ 61

3. Fill in the missing number in each of the following.

   a. $\square \div (-3) = -9$

   b. $(+36) \div (-4) = \square$

   c. $(-22) \div \square = +11$

   d. $(-48) \div \square = -8$

   e. $(+24) \div (-6) = \square$

4. Find the quotient for each of the following.

   a. $(+70) \div (-10) = \square$

   b. $(-36) \div (+12) = \square$

   c. $(-48) \div (-6) = \square$

   d. $(+18) \div (+3) = \square$

   e. $(+35) \div (-7) = \square$

5. Solve this word problem by using a number sentence.

   A diver descends from the surface to a depth of -100 m in five minutes. How many metres did the diver descend in one minute?

See your learning facilitator to check your answers and to receive further instructions.

Space for Your Work

Working Together

The following examples involve problem-solving situations that involve division of integers. Number lines are included to help you picture the events and the calculations.

Example 1

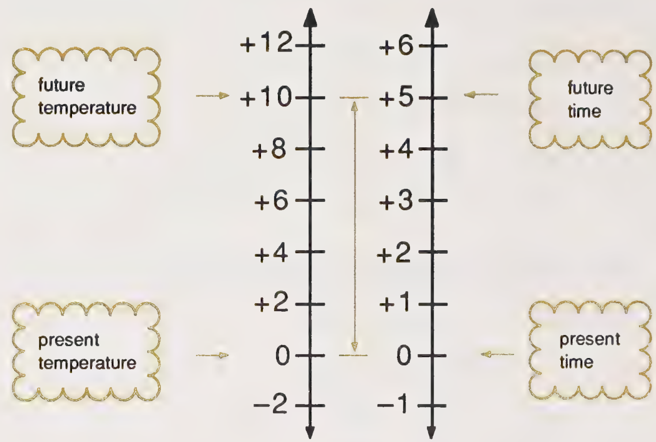
The present temperature is 0°C. If the temperature rises 2°C each hour, how long will it take to reach + 10°C?

The event can be described by the following number sentence.

$$(+10) \div (+2) = \blacksquare$$

future temperature | increase per hour | time

The event can be pictured by using a number line.



The mental calculation is as follows.

$$(+10) \div (+2) = +5$$

10 ÷ 2

The signs are like, so the quotient is positive.

It will be + 10°C in five hours or at + 5 hours.

Example 2

The present temperature is 0°C. If the temperature is rising 2°C each hour, how long ago was it – 8°C?

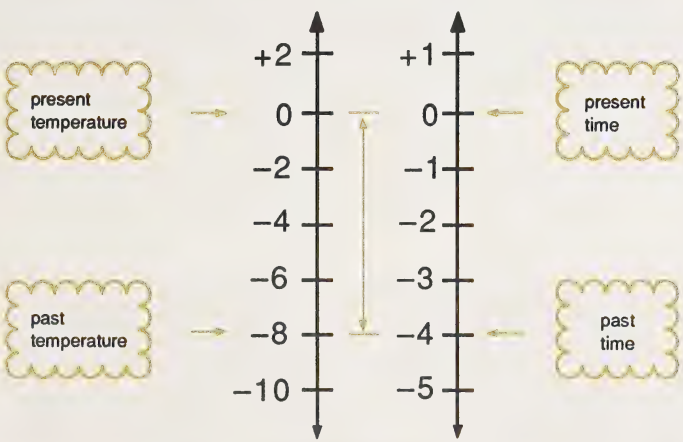
Solution

Here is a number sentence describing the event.

$$(-8) \div (+2) = \blacksquare$$

past temperature | increase per hour | time

Here is a number line to picture the event.



The mental calculation is as follows.

$$(-8) \div (+2) = -4$$

8 ÷ 2

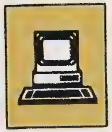
The signs are unlike, so the quotient is negative.

It was – 8°C four hours ago or at – 4 hours.

Extra Practice

Space for Your Work

Computer Alternative

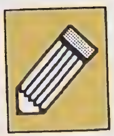


1. Use the program *Integers* from the disk *Integers/Integer Fast Facts* (EduSoft).

   Read the User's Guide. Choose the multiplication operation.

Print Alternative

Use number lines to help you complete Questions 2 and 3.



2. The present temperature is 0°C. If the temperature is falling 2°C each hour, how long will it take to reach – 6°C?

   The number sentence describing the event is this.

$$(-6) \div (-2) = ▩$$

future temperature | decrease per hour | time

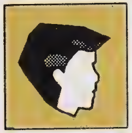
3. The present temperature is 0°C. If the temperature is falling 2°C each hour, how long ago was it + 8°C?

The number sentence describing the event is this.

$$\underbrace{(+8)}_{\text{past temperature}} \div \underbrace{(-2)}_{\text{decrease per hour}} = \underbrace{▒}_{\text{time}}$$

Space for Your Work

See your learning facilitator to check your answers and to receive further instructions.



Working Together

Do you remember the Droopy model from Section I2, where you studied multiplying integers?

Droopy also tackles multiplication of integers by the following rules.

- He always starts on the zero coordinate on the number line.
- He has two sweaters, one with a positive sign (+), the other with a negative sign (–) on it. When Droopy wears his positive sweater, he faces in the positive direction on the number line. When Droopy wears his negative sweater, he faces in the negative direction on the number line.
- The sign of the first factor determines which sweater he will wear and thus the way he will face. If the sign of the first factor is positive he wears his positive sweater and faces to your right.

If the sign of the first factor is negative, Droopy puts on his negative sweater and faces in the negative direction on the number line, that is to your left.

- The sign of the second factor determines which way Droopy moves. If the sign of the second factor is positve, he moves forward. If the sign of the second factor is negative, he moves backward.

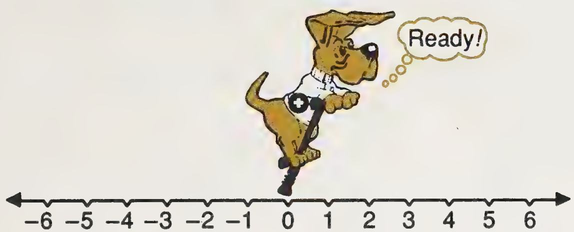
You can use the Droopy model to help you find the quotient of integers, too.

Example 1: What is $(+6) \div (+2)$?

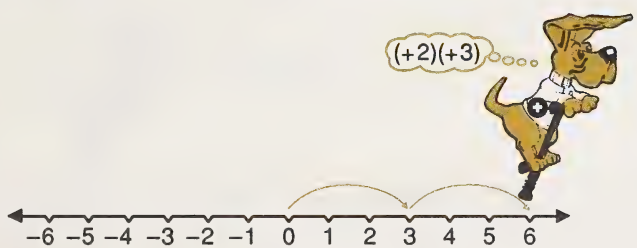
Solution

$(+6) \div (+2) = \blacksquare$ means the same as
$(+2) \times \blacksquare = +6$.

Since the first factor is positive, place the positive Droopy at zero facing right.



In order for Droopy to get to + 6, he must move forward in two steps of 3.



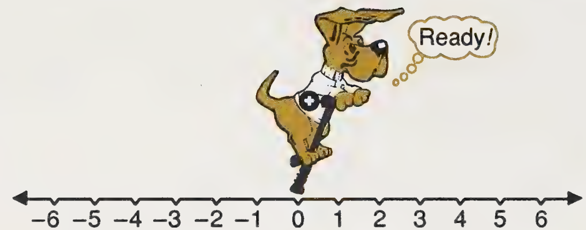
So, $(+6) \div (+2) = +3$.

Example 2: What is $(-6) \div (+2)$?

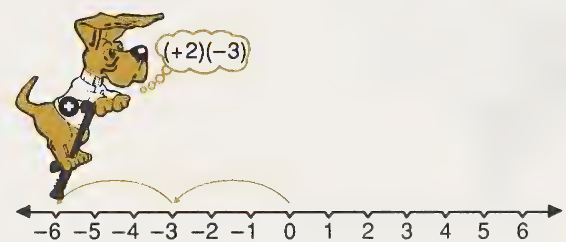
Solution

$(-6) \div (+2) = \blacksquare$ means the same as
$(+2) \times \blacksquare = -6$.

Since the first factor is positive, place the positive Droopy at zero facing right.



In order for Droopy to get to – 6, he must move backward in steps of 3.



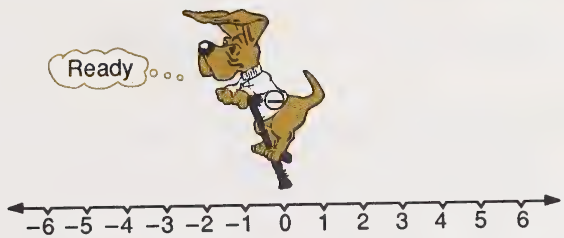
So, $(-6) \div (+2) = -3$.

Example 3: What is $(-6) \div (-2)$?

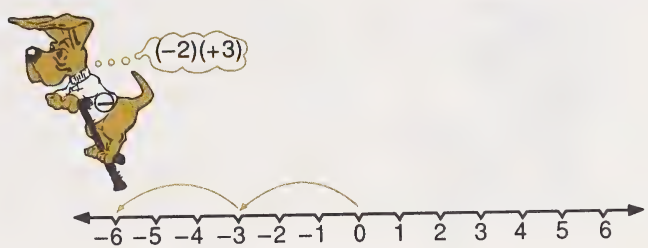
Solution

$(-6) \div (-2) = \blacksquare$ means the same as
$(-2) \times \blacksquare = -6$.

Since the first factor is negative, place the negative Droopy at zero facing left.



In order for Droopy to get to -6, he must move forward in two steps of 3.



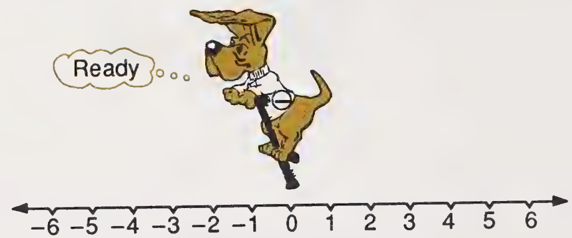
So, $(-6) \div (-2) = +3$.

Example 4: What is $(+6) \div (-2)$?

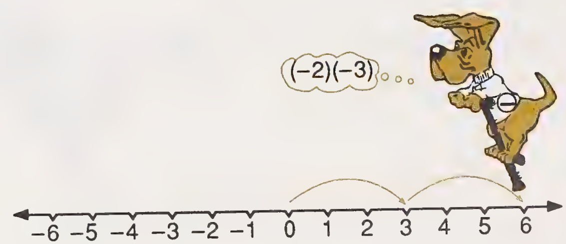
Solution

$(+6) \div (-2) = \blacksquare$ means the same as
$(-2) \times \blacksquare = +6$.

Since the first factor is negative, place the negative Droopy at zero facing left.



In order for Droopy to get to $+6$, he must move backward in two steps of 3.



So, $(+6) \div (-2) = -3$.

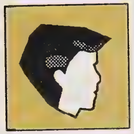
Further Practice

Space for Your Work

Find the quotient. If you wish, use the cutout of Droopy and the number lines to help you.

1. $(+8) \div (+4) = \blacksquare$
2. $(+8) \div (-4) = \blacksquare$
3. $(-8) \div (+4) = \blacksquare$
4. $(-8) \div (-4) = \blacksquare$
5. $(+15) \div (-5) = \blacksquare$
6. $(-9) \div (-3) = \blacksquare$
7. $(-16) \div (+4) = \blacksquare$

See your learning facilitator to check your answers and to receive further instructions.



Working Together

Your calculator can be used to divide integers. You will need to use the sign change key [+/−] as you did when multiplying integers.

Example 1: What is $(+8) \div (+4)$?

Solution

Key Press	Display
8	8
÷ 4	4
=	2

So, $(+8) \div (+4) = +2$.

Example 2: $(-8) \div (-4)$?

Solution

Key Press	Display
8 +/−	−8
÷ 4 +/−	−4
=	2

So, $(-8) \div (-4) = +2$.

Example 3: What is $(+8) \div (-4)$?

Solution

Key Press	Display
8	8
÷ 4 +/−	−4
=	−2

So, $(+8) \div (-4) = -2$.

Example 4: What is $(-8) \div (+4)$?

Solution

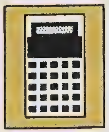
Key Press	Display
8 +/−	−8
÷ 4	4
=	−2

So, $(-8) \div (+4) = -2$.

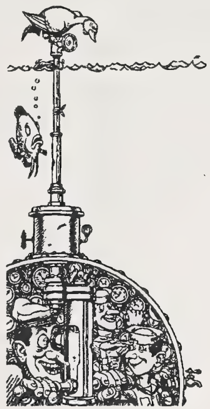
Concluding Activities

Space for Your Work

Use your calculator to complete Questions 1 and 2.



1. A submarine descends 30 m every minute. How long will it take to descend from sea level to – 1 800 m?



2. Complete the following puzzle[1].

See your learning facilitator to check your answers and to receive further instructions.

[1] Creative Publications for excerpts from *Algebra with Pizzazz*.

What Did ZORNA Say When She Married a 3-foot Pygmy?

Do each of the exercises to the right and find your answer in one of the boxes at the bottom of the page. Write the letter of the exercise in that box. The answers are arranged in order from smallest to largest. Keep working and you will discover the answer to the title question.

A $(-12) \div (+4)$
E $(+60) \div (+15)$
T $(+45) \div (-9)$
A $(-48) \div (-4)$
R $(-49) \div (-7)$
A $(-3) \div (-3)$
E $(-60) \div (+5)$
O $(-200) \div (+4)$
A $(-90) \div (+9)$
H $0 \div (-7)$
D $(+77) \div (-7)$
E $(-215 \div (+1)$
T $(+96) \div (+12)$
E $(-75) \div (-5)$
O $(+56) \div (-8)$

A $(+750) \div (+10)$
E $(-42) \div (-7)$
R $(-150) \div (+2)$
E $(-100) \div (-2)$
T $(+67) \div (-1)$
N $(-80) \div (-40)$
H $(+150) \div (-5)$
R $(-30) \div (+5)$
T $(+1700) \div (-10)$
V $(+100) \div (+20)$
T $(+13) \div (-13)$
V $(+120) \div (+4)$
M $(-100) \div (+25)$
V $(-42) \div (+3)$
L $(+80) \div (+5)$

E $\frac{(+36)}{(-2)}$
O $\frac{(-50)}{(-2)}$
A $\frac{(+100)}{(-4)}$
D $\frac{(-670)}{(-10)}$
E $\frac{(+9100)}{(-100)}$
O $\frac{(+45)}{(+3)}$
A $\frac{(+600)}{(+4)}$
V $\frac{(+39)}{(+3)}$
O $\frac{(-54)}{(-6)}$
L $\frac{(+311)}{(+1)}$
N $\frac{(+38)}{(-19)}$

V $\frac{(-63)}{(+3)}$
T $\frac{(+300)}{(-2)}$
H $\frac{(+1000)}{(+100)}$
B $\frac{(+3110)}{(-10)}$
N $\frac{(+900)}{(+300)}$
S $\frac{(+81)}{(-9)}$
L $\frac{(-430)}{(-2)}$
H $\frac{(-48)}{(+6)}$
L $\frac{(-48)}{(+3)}$
T $\frac{(-91)}{(-1)}$

−311	−215	−170	−150	−91	−75	−67	−50	−30	−25	−21	−18	−16	−15	−14	−12	−11	−10
−9	−8	−7	−6	−5	−4	−3	−2	−1	0	1	2	3	4	5	6	7	
8	9	10	12	13	15	16	25	30	50	67	75	91	150	215	311		

ORDER OF OPERATIONS

What Lies Ahead

In this section you will learn these skills.

- identifying the correct order of operations for integers
- using order of operations correctly when calculating with integers

Working Together

When dealing with a series of operations, you need to apply the rules concerning order of operations.

You have already used order of operations with whole numbers. You should recall the order in which operations are to be done.

- Operations in brackets
- Exponents
- Divide and multiply from left to right. Do whichever operation is on the left first.
- Add and subtract from left to right. Do whichever operation is on the left first.

Remember **BEDMAS**. This mnemonic helps you remember the order of operations.

B → brackets
E → exponents
D/M → division/multiplication
A/S → addition/subtraction

The same order of operations rules apply when working with integers.

Example 1: Simplify $(-5) \times (-3) \div (+5) \times (-2)$.

Solution

$$\begin{aligned}
&(-5) \times (-3) \div (+5) \times (-2) \\
&= (+15) \div (+5) \times (-2) \\
&= (+3) \times (-2) \\
&= -6
\end{aligned}$$

Example 2: Simplify $(-5) + (-3) - (-2)$.

Solution

$$\begin{aligned}
&(-5) + (-3) - (-2) \\
&= (-8) - (-2) \\
&= (-8) + (+2) \quad \leftarrow \text{change to addition} \\
&= -6
\end{aligned}$$

Example 3: Simplify $(-5) \times (-3) + (+4) \div (-2)$.

Solution

$$\begin{aligned}
&(-5) \times (-3) + (+4) \div (-2) \\
&= (+15) + (+4) \div (-2) \\
&= (+15) + (-2) \\
&= +13
\end{aligned}$$

Example 4: Simplify $[(+4) - (-3)] - [(+4) + (-2)] \times (-6) \times (-1)$.

Solution

$$[(+4) - (-3)] - [(+4) + (-2)] \times (-6) \times (-1)$$

$= (+7) - (+2) \times (-6) \times (-1)$

$= (+7) - (-12) \times (-1)$

$= (+7) - (+12)$

$= (+7) + (-12)$ ← change to addition

$= -5$

Example 5: Simplify $\dfrac{(-5) + (+2) \times (+1)}{(-2) \times (-1) - (+1)}$.

Solution

$\dfrac{(-5) + (+2) \times (+1)}{(-2) \times (-1) - (+1)}$ ← division

$= [(-5) + (+2) \times (+1)] \div [(-2) \times (-1) - (+1)]$

$= [(-15) + (+2) \times (+1)] \div [(-2) \times (-1) - (+1)]$

$= [(-5) + (+2)] \div [(+2) - (+1)]$

$= (-3) \div [(+2) + (-1)]$ ← change to addition

$= (-3) \div (+1)$

$= -3$

Practice Activities

Space for Your Work

1. Simplify each of the following.

   a. $(-2) \times (+3) - (-2)$

   b. $(+7) - (+8) \times (-3)$

   c. $(-4) - (+4) \div (-2)$

   d. $(+9) \div (-3) + (+5) \times (-2) - (+1)$

2. Simplify each of the following.

   a. $(-4) \times (-3) \div [(+2) + (+2)]$

   b. $[(+6) - (-4)] \times (-12) \div (-6)$

   c. $(-3) \times (+5) - [(-42) \div (+7) - (-3)]$

   d. $[(+13) + (-11)] - (+1) \times (-4) + [(+19) \times (+23)]$

   e. $(+99) - [(+4) + (-10) \div (+5) \times (-8)]$

3. Sanchez likes golf. Over one season he played 200 games. In 85 games he was three under par or -3. In 60 games he was one under par or -1. In 30 games he was two over par or $+2$, and in 25 games he was three over par or $+3$. Calculate how much over or under par he was for the 200 games played in one season.

Space for Your Work

See your learning facilitator to check your answers and to receive further instructions.

Extra Practice

Space for Your Work

1. Simplify each of the following.

   a. $(+3) - (-4) + (-2)$

   b. $(-2) \times (-6) \times (-3)$

   c. $(+4) + (-2) \times (-5)$

   d. $(+14) \div (-2) + (+6) \times (-3)$

2. Simplify each of the following.

   a. $[(-4) + (-2)] - [(-6) \times (+3)]$

   b. $(+3) - [(-2) - (-4)]$

   c. $(+2) \times [(+1) + (-5)] \div (-4)$

   d. $[(-18) \div (-9) \times (+4)] - [(+14) - (+4)] \div (-2)$

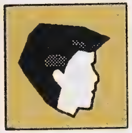
3. Sasha had several types of stocks. At the end of the year she had to calculate her total profit or loss on the stocks. The table below shows the number of shares in each stock and the profit or loss per share.

Name	Number of Shares	Profit (+) or Loss (–) on Each Share
Happy Helicopters	50	+ 2
Duggit Mines	75	– 3
Black Earth Petroleum	100	+ 1
New Wave Hair Salons	125	– 4

Calculate Sasha's total profit or loss for the year.

Space for Your Work

See your learning facilitator to check your answers and to receive further instructions.



Working Together

You can evaluate a series of operations on a calculator. However, not all calculators work the same way. Some calculators allow you to press the keys in the order the series is written. Other calculators require you to use the rules for order of operations and to press the keys in the same order as when you use paper and pencil. Consider the following example. Example 1 uses whole numbers. Example 2 uses integers.

Example 1: What is $7 + 6 \times 3$?

Solution

When using paper and pencil this is the result.

$$\begin{aligned} & 7 + 6 \times 3 \\ = {} & 7 + 18 \\ = {} & 25 \end{aligned}$$

Multiplication is done before addition.

When using a regular calculator this is the result.

Key Press	Display
6 x 3 =	18
+ 7 =	25

When using a scientific calculator this is the result.

Key Press	Display
7 + 6 x 3 =	25

Example 2: What is $(+2) + (-3) \times (+4)$?

Solution

When using paper and pencil this is the result.

$$\begin{aligned} & (+2) + (-3) \times (+4) \\ = {} & (+2) + (-12) \\ = {} & -10 \end{aligned}$$

Multiplication is done before addition.

When using a regular calculator that has a +/- key this is the result.

Key Press	Display
3 +/- x 4	−12
+ 2 =	−10

When using a scientific calculator this is the result.

Key Press	Display
2 + 3 +/- x 4 =	−10

A calculator's memory can help you evaluate a series of operations. The memory on a calculator allows you to store numbers in the calculator's memory while you work with other numbers. Four keys can be used.

- **M+** adds the number in the display to the number in the calculator's memory.
- **M–** subtracts the number in the display from the number in the calculator's memory.
- **MR** recalls the number in the memory to the display.
- **MC** clears the number in the memory.

Example 1: What is $7 + 6 \times 3$?

Solution

- 7 is placed in the memory.
- Next, the product of 6 and 3 is added to the number in the memory.
- Finally, the number in the memory is recalled to the display.

Key Press	Display
7 M+	M 7
6 x 3 = M+	M 18
MR	M 25

So, $7 + 6 \times 3 = 25$.

This can be done on most calculators. If the one you are using does something different, see your calculator manual.

Example 2: What is $9 \times 6 - 4 \times 3$?

Solution

- The product of 9 and 6 is placed in the memory.
- The product of 4 and 3 is subtracted from the memory.
- Finally, the number in the memory is recalled to the display.

Key Press	Display
9 x 6 M+	M 54
4 x 3 M–	M 12
MR	M 42

So, $9 \times 6 - 4 \times 3 = 42$.

Example 3: What is $(+2) + (-3) \times (+4)$?

Solution

- 2 is placed in the memory.
- Next, the product of -3 and $+4$ is added to the number in the memory.
- Finally, the number in the memory is recalled.

Key Press	Display
2 M+	M 2
3 +/– × 4 M+	M –12
MR	M –10

So, $(+2) + (-3) \times (+4) = -10$.

Example 4: What is $(+4) - (-3) \times (-12)$?

Solution

- 4 is placed in the memory.
- Next, the product of -3 and $+2$ is subtracted from the memory.
- Finally, the number in the memory is recalled to the display.

Key Press	Display
4 M+	M 4
3 +/– × 2 M+	M 6
MR	M –2

So, $(+4) - (-3) \times (-2) = -2$.

Usually you will not have to use the memory of a scientific calculator to perform a series of operations because scientific calculators allow you to press the keys in the order that the expression is written in. Unless you have a bracket key on your scientific calculator, you will not be able to press the keys in the order the expression is written when brackets are involved. You will have to use the memory then. See the following examples.

Example 1: What is $7 \times (5 - 3)$?

Solution

When using paper and pencil, this is the result.

$$7 \times (5 - 3)$$
$$= 7 \times 2$$
$$= 14$$

Operations in brackets are performed first.

When using a calculator, this is the result.

Key Press	Display
5 − 3 =	2
x 7 =	14

Example 2: What is $(+3) \times [(-2) - (-3)]$?

Solution

When using paper and pencil, this is the result.

$$(+3) \times [(-2) - (-3)]$$
$$= (+3) \times [(-2) + (+3)]$$
$$= (+3) \times (+1)$$
$$= +3$$

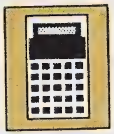
Operations in brackets are performed first.

When using a calculator that has a +/− key, this is the result.

Key Press	Display
2 +/− − 3 +/− =	1
x 3 =	3

Concluding Activities

Space for Your Work



1. Evaluate the following on paper first. Then use your calculator to see if you can get the same result. Remember to clear the calculator display before computing. Press $\boxed{C}$ to clear the display.

   a. $49 + 200 \div 25$

   b. $12 - 18 \div 9 + 4$

   c. $(+41) + (-512) \div (-16)$

   d. $(-28) + (+4) \div (-2) - (-32)$

2. Evaluate the following on paper first. Then use your calculator to see if you get the same result. Remember to clear the memory after each question by pressing $\boxed{\text{MC}}$.

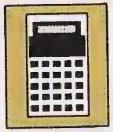
a. $18 + 72 \div 6 + 24$

b. $17 \times 11 + 5 \times 18$

c. $(-2) + (+8) \times (-3)$

d. $(-5) - (+7) \times (-2)$

Space for Your Work



3. Evaluate the following on paper first. Then use your calculator to see if you can get the same result. Remember to clear the memory after each question by pressing $\boxed{\text{MC}}$.

   a. $5 + 3 \times (9 - 2)$

   b. $(3 + 5 - 2) \div 6$

   c. $[(-150) + (+6)] \div 12$

   d. $(+13) + [(-2) + (-10)]$

Space for Your Work

See your learning facilitator to check your answers and to receive further instructions.

SUMMARY

What Lies Ahead

In this section you will review these skills.

- adding integers
- subtracting integers
- multiplying integers
- dividing integers
- performing a series of operations

Working Together

At this point it is a good idea to review the skills you have learned in Sections 8 to 14.

Turn to Section 8 and review the pretest. Correct any errors you may have made. You may be pleasantly surprised to discover how much you have learned.

MODULE CONCLUSION

What Lies Ahead

In the Module Conclusion you will do the Module Assignment.

Working Together

Now that you have finished the work in this booklet you should be ready for the Module Assignment.

Module Assignment

Turn to the Assignment Booklet and do the Module Assignment. You may use your notes, but do the assignment independently.

Afterwards, submit the assignment for a grade and feedback from your learning facilitator.

APPENDIX

GLOSSARY

Absolute value: the distance a number is from zero without indicating the direction

$|+3| = 3$, $|-3| = 3$

Additive inverses: two numbers whose sum is zero

For example, + 2 and – 2 are additive inverses.

Base: the number being multiplied by itself when it is part of a power

3^5 ← base

Common multiple: a number that is a multiple of two or more numbers

6 is a common multiple of 2, 4, and 8.

Common factor: a number that is a factor of two or more numbers

3 is the common factor of 15 and 21.

Composite number: a whole number with more than two factors

10 is a composite number. Its factors are 1, 2 and 5.

Cube of a number: the third power of a number

5^3 or 125 is the the cube of 5.

↑

Read as "five cubed".

Expanded form: a form of a number written as the sum of the products of each digit and its place value

$$326 = (3 \times 100) + (2 \times 10) + (6 \times 1)$$

$$\text{or } 326 = (3 \times 10^2) + (2 \times 10^1) + (6 \times 1)$$

Exponent: a number showing how many times the base is used as a factor

3^5 ← exponent

Exponential form: a form of a number written as a power

Factor (noun): any one of the numbers used in multiplication to form a product

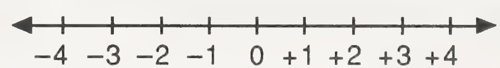
In $3 \times 2 \times 5 = 30$, the 3, 2, and 5 are factors of 30.

Factor (verb): to express a whole number as a product of its factors

Greatest common factor (GCF): the greatest factor common to two or more numbers

The GCF of 12 and 18 is 6.

Integers: all the whole numbers and their opposites



Least common multiple (LCM): the least of the non-zero multiples common to two or more numbers

30 is the LCM of 15 and 6.

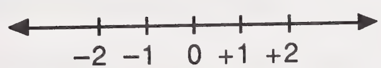
Multiple: the product of a number and another whole number

The multiples of 3 are 0, 3, 6, 9, 12, ...

Negative integer: a number less than zero

Number line: a line used to show a sequence of numbers

Opposite integers: integers that are equidistant from zero on the number line but in opposite directions



– 2 and + 2 are opposite integers.

Opposite operations: operations which undo each other

+ and – are opposite operations.

× and ÷ are opposite operations.

Positive integers: integers greater than zero

Power: a product of equal factors

$3^4 = 3 \times 3 \times 3 \times 3 = 81$ is the fourth power of 3.

This may be read as 3 exponent 4.

Powers of 10: the powers with bases of 10

10^1 or 10, 10^2 or 100, 10^3 or 1000, ...

Prime number: a number that has exactly two factors: itself and 1

2, 3, 5, 7, and 11 are prime numbers.

Prime factor: a factor that is a prime number

Prime factorization: the expression of a number as the product of its prime numbers

Proper factor: a factor that is less than the number it is a factor of

The proper factors of 6 are 1, 2, and 3. 6 is not a proper factor of 6.

Relatively prime: having a greatest common factor of 1

6 and 7 are relatively prime.

Scientific notation: a notation for writing a whole number as a product of a number between 1 and 10 and a power of 10

$$32\,000\,000\,000\,000 = 3.2 \times 10^{13}$$

Square of a number: the second power of a number

3^2 or 9 is the square of 3.

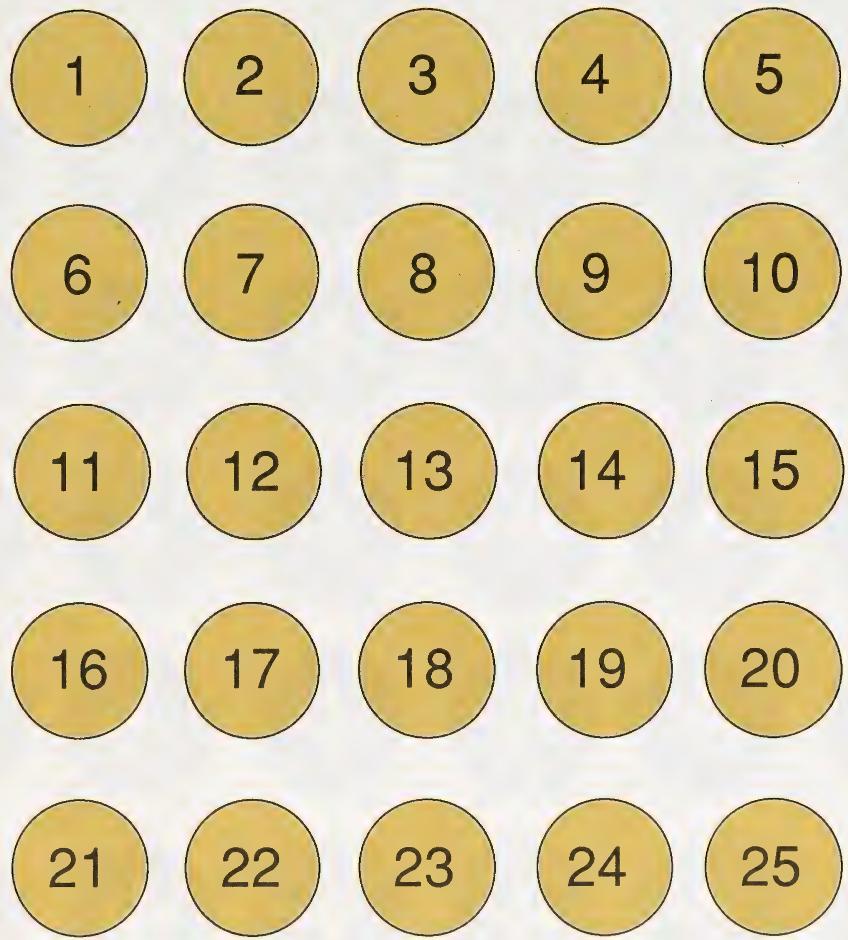
Read as "three squared".

Standard form: the usual form of a numeral

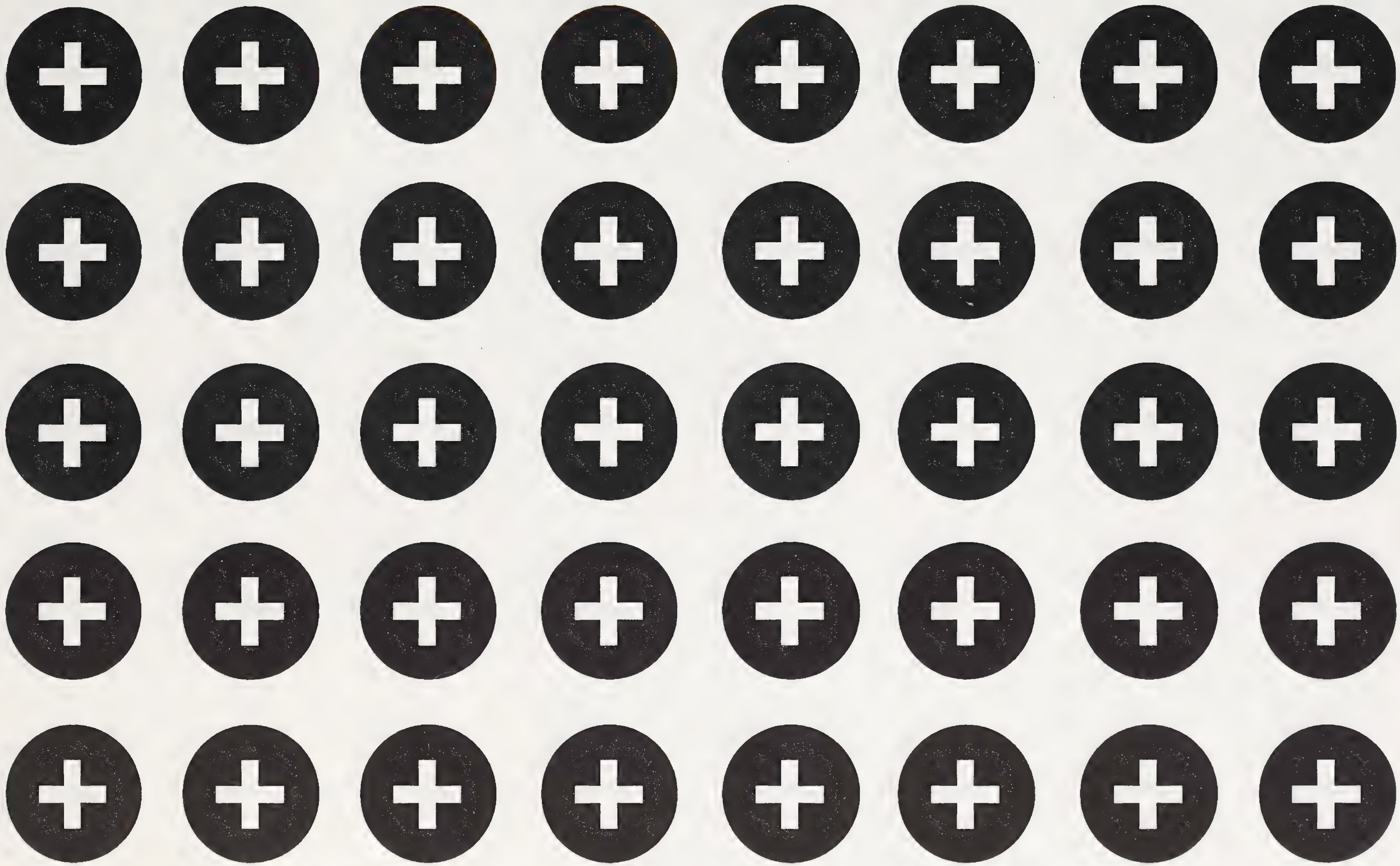
MULTIPLE BOARDS

1	2	3	4	5	6	7
2	4	6	8	10	12	14
3	6	9	12	15	18	21
4	8	12	16	20	24	28
5	10	15	20	25	30	35
6	12	18	24	30	36	42

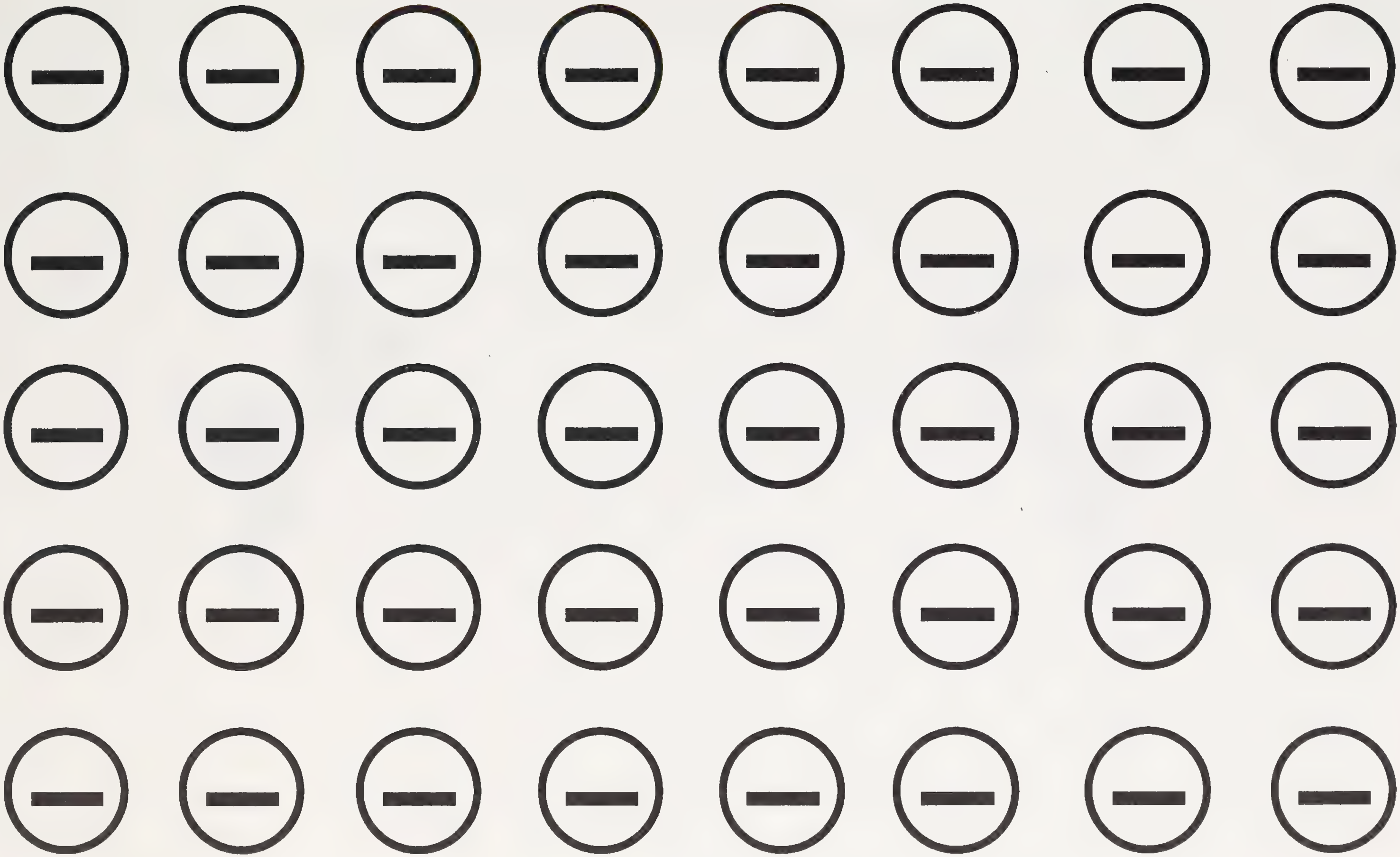
COINS FOR *FACTORS* GAME



POSITIVE COUNTERS



NEGATIVE COUNTERS



DROOPY

PLACE ROLL

5	3	7	PRIME
2	6	8	17
PRIME	21	4	16
13	9	PRIME	26

THE LCM GAME

12	20	6	30
9	4	15	3
1	10	4	12
6	30	20	2

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